Scottish Journal of Political Economy, Vol. 54, No. 3, July 2007

RATIONAL EXPECTATIONS AND PRO SPORTS LEAGUES

Rodney Fort and James Quirk***

Abstract

We put a new set of shoes on that old workhorse, the competitive talent market (CTM) model in sports economics. There exist unique rational expectations equilibria for both national football league (NFL-type leagues) and major league baseball (MLB-type leagues) under the CTM model. A cursory statistical test fails to reject the empirical implications for the NFL-type league. The model also suggests empirical tests of whether or not talent demand (marginal revenues from talent), including induced effects, actually slopes down. But like all models, the competitive talent model should be applied in its context. It describes highly cooperative North American sports leagues that have a wealth of common information. But it may not do the same for other leagues if they lack this common information.

I Introduction

The competitive talent market (CTM) model is an old work horse in sports economics (El-Hodiri and Quirk, 1971; Quirk and El Hodiri, 1974; Fort and Quirk, 1995; Vrooman, 1995; recent examples are in Szymanski, 2004; Szymanski and Kesenne, 2004; Kesenne, 2005). In this paper, we give it a new pair of shoes. There exist unique rational expectations (RE) equilibria for the CMT model. We show this for leagues with two distinct revenue functions, one characterizing national football league (NFL-type leagues), the other major league baseball (MLB-type leagues), both without and with modern pooled revenue sharing in each case.

Of course, uniqueness seldom comes for free and our results are no exception. In general, uniqueness can be proved under the following reasonable assumption – the direct impact of a team's talent choice on its own marginal revenue is greater than the indirect impacts on its marginal revenue associated with or induced by talent choice responses by other teams. We call this the 'dominant direct effects' assumption. It ends up that this effect is implicitly assumed in all

*University of Michigan
**California Institute of Technology (Retired)

past work based on the CTM where talent demand, including induced effects, slopes downward. This suggests that tests of this assumption are in order.

A preview of our findings is as follows. First, RE in the CTM model takes care of the theoretical possibility of informational externalities. Second, RE in the CTM model moves interpretation of the CTM away from *how* the equilibrium process needs to function (e.g., Walras' tatonnement as in El Hodiri and Quirk, 1971; Quirk and El Hodiri, 1974) and toward what that process actually produces. Third, RE lends theoretical underpinnings to observations about the free flow of information in North American pro sports leagues. In particular, North American pro sports leagues seem completely likely candidates for the CMT model because the information prerequisites for RE exist in the first place. Finally, empirically testable propositions are suggested that have the potential to reject this RE theory.

The paper proceeds as follows. In Section II, we discuss sports leagues from the CTM and RE perspective. In Section III, we present our RE findings for the NFL-type league. We do the same for the MLB-type league in Section IV. The implications for future modeling practice are collected in Section V and conclusions round out the paper in Section VI.

II CTM AND RE

Arguing for RE in sports leagues hinges on whether individual team owners and other participants (e.g., players, agents, etc.) (1) recognize all of the information that is available to them, (2) organize themselves in such a way as to take full advantage of all that information has to offer concerning their economic welfare, and (3) act under the common knowledge assumption that other actors will respond rationally and predictably to the information available. In addition, technically speaking, RE rests on two components – each person's behavior can be described as the outcome of maximizing an objective function subject to perceived constraints and the constraints perceived by everybody in the system are mutually consistent.

In finance and macroeconomics, this idea is used in various forms to imply that all information available to all decision makers is used efficiently; there can be no systematic forecast errors. Along these lines, we discuss here just why the CTM/RE explanation of North American leagues (NALs) fits the bill before formal modeling in succeeding sections.

If talent is measured so that a unit increase in talent increases the winning percent by one unit, then the talent market is also the market for the teams' winning percents, that is, their long-run quality decisions. The league members know well that each of their talent choices spill over to the rest and that competitive balance will be an issue with fans. But just how to handle these issues requires institutional structure. And that is one function of the entity called 'the league,' namely, to reach consensus on institutional arrangements among its member owners. We have no doubt that, at times, this small-government process may be characterized by non-cooperative behavior as all league choices have distributional consequences. But this is the realm of public choice analysis of league decision-making processes, not of the CTM model and RE in the talent market. And even in the league decision-making process, there is plenty of information well known to all.

We do not mean to gloss over the presence of inter-team talent choice externalities of the variety first noted by Canes (1974). Indeed, adding RE to the CMT model can overcome the fact that a Walrasian tatonnement process sidesteps the externality issue. Suppose that each owner submits a bid for a winning percent, assuming that the announced price of talent is an equilibrium price. Furthermore, suppose that the owner's projections of the winning percents chosen by other owners are correct. If the conjectures of different owners are not consistent, the referee announces a new potential equilibrium price of talent. When all these conjectures are consistent with one another, we have the Walrasian equilibrium. Hence, without any RE component, the CMT has a tatonnement referee take care of the externality issue.

Turning to an RE approach resolves this informational externality issue directly. Put simply, under RE, everyone can calculate the RE equilibrium so why would any participant in the market, during an adjustment process of any kind, accept anything less than can be achieved at the RE equilibrium? Quite aside from the theoretical derivations in subsequent sections, we argue that the CMT model works well in NALs because the informational environment is such that an RE equilibrium is achieved under competitive conditions. This happens regardless of the adjustment mechanism proposed as rational behavior by participants insures that the only transactions that will take place will be equilibrium transactions.

Interestingly, there is experimental evidence that this actually does happen. In a number of experimental studies involving two-sided auctions, the first few plays see prices that violate the Walrasian equilibrium. But after repeated play, as knowledge of the auction is accumulated, even in the absence of a Walrasian referee, there is strong evidence of rapid convergence to a market-clearing Walrasian equilibrium with few if any non-equilibrium trades (Holt, 2006).

There are stiff information requirements for RE but we can think of no other industry that fits the bill better than NALs. No other industry has so much information about production functions, revenue streams, supply conditions, and the like concerning competitors available to any owner in the industry. Indeed, those practicing sports economics often begin papers by pointing out just this reason why sports are such a fertile ground for analyzing economic questions. There really is common knowledge of just about everything in sight, except possibly the motivation of owners. And even that aspect is taken care of if we restrict ourselves to a world in which all owners maximize profit.

This information availability is the result of at least three factors. First, fan and media fascination with the performance of athletes and teams appears unlimited. We are deluged with minute-by-minute measures of performance in timed sports (NBA, NFL, and NHL). In MLB, an entire cadre of sophisticated statistical analysts searches for the holy grail of statistical descriptions to capture the elements of player performance.

Second, there are no other options at the major league level outside of the leagues themselves. Owners turn inward in the pursuit of economic welfare; what is the point of any secrecy when there is no rival league to turn to? The meat of off-season league meetings is just this type of information sharing for the good of the league.

Finally, institutional arrangements necessitate essentially complete information sharing. The administration of revenue-sharing arrangements, payroll taxes, and salary caps requires full reporting to the league office by all member owners. Labor law further requires reporting of this type of information to the officers of player unions. And additional information cooperation is also required in actual collective bargaining agreements. It has also been documented that the result of competition policy for North American leagues actually has been to encourage this cooperation, even if it is detrimental to sports fans (Quirk and Fort, 1999; Fort, 2000, 2007). All in all, members of sports leagues recognize all of the information that is available to them and organize themselves to take full advantage of all that information has to offer concerning their economic welfare.

Even those rare transactions occurring at non-equilibrium prices are correctible, without affecting the RE equilibrium, through subsequent trades. During the trading process, teams that discover they have over-invested in winning percent can reenter the market to correct for any perceived mistakes. Furthermore, as the major part of the trading process occurs in the interim between league seasons, those purchases and sales do not shift the underlying demand and supply functions as is the case, say, with consumption goods where there is no resale market.

For NALs, there is enough shared information on private revenue and cost to make RE a realistic portrayal. And the institutional structure of leagues has developed in an effort to minimize the possibility that actions by any one owner could harm the profitability of the league as a whole. At least for NALs, internally enforceable rules guarantee that essentially all information relevant to decision making is in fact 'common knowledge' for every owner in the league. Hence, with all general managers acting alike, under 'common knowledge,' we get every owner correctly taking into account the effect of their decisions on every other owner in the league. Decisions by general managers are all jointly consistent with one another as, in all leagues, the sum of winning percents must equal half the number of teams (the well-known adding-up constraint).

If the common information arguments just listed are convincing, then all that has to be assumed is profit maximization in order to generate an RE equilibrium, one that can be calculated by any and all general managers. As everyone knows the allocation of talent that will occur at the RE equilibrium and the RE price of talent, why would any team pay more than that price for a unit of talent, and why would any team demand less than this? Hence the equilibrium price gets established and, at that equilibrium price, the allocation of talent is also determined. That allocation, on the one hand, is the profit-maximizing allocation for any owner assuming every other owner is at their RE equilibrium allocation and, on the other hand, clears the talent market.

Strictly speaking, we present no dynamic process to obtain an RE equilibrium. But with all of the information common knowledge to all owners, any observed price in the market for players is going to be close to the RE equilibrium price. This remaining obstacle, to be leapt over in moving to the RE equilibrium, does not seem insurmountable.

III RE and the NFL-Type League

Formally, in what follows, an RE equilibrium (w^*, p^*) is a vector w^* of choices by actors and a price p^* such that all the following hold. The choice of any actor maximizes their objective function given both the choices of other actors and p^*; all such choices are feasible and consistent with one another; and the resulting vector w^* is a market-clearing vector, given p^*.

In our first case, revenue functions are dominated by season ticket sales. This seems to us to characterize leagues like the NFL. With eight home games and two pre-season games to sell (in contrast to, say, MLB's 81 home games, detailed in the next section), and average ticket prices around $100, this suggests a season ticket price around $900. If their team performs below expectations, fans have only lost the value of the remaining games they then choose not to attend. And when the team does perform up to expectations, in terms of post-season chances, every game in the NFL is more important to fans, even those against poorer opponents. This reduces the importance of visiting team quality in the fan purchase decision. Hence football owners are able to transfer the risk that the team performs below expectations to fans. Fans confronted primarily with season ticket options must make their estimate of the value of that purchase primarily on the quality of the home team.

Other than common knowledge detailed in the last section, we assume that the choice of talent and the choice of winning percent are the same thing. And the usual adding-up constraint for league play also holds, that is, $\sum_{i=1}^{n} w_i = \frac{n}{2}$, where w_i is the winning percent of team i and n is the number of teams in the league.

Without any revenue sharing, with strictly concave revenues R_i dependent only on the team's own winning percent, w_i, and with the price of talent (and winning percent as well) denoted p, profits are

$$\pi_i = R_i(w_i) - pw_i, \quad i = 1, \ldots, n. \tag{1}$$

The first-order conditions are

$$\frac{d\pi_i}{dw_i} = \frac{dR_i}{dw_i} - p = MR_i - p = 0, \quad i = 1, \ldots, n. \tag{2}$$

Rearranging equation (2) shows that winning percent is chosen until marginal revenue equals the price of talent

$$MR_i = p, \quad i = 1, \ldots, n. \tag{3}$$

It also is clear from expression (3) that equilibrium must have $MR_i = MR_j$, for all i, j.

Let (w*, p*) be an RE equilibrium in this case. Then, following our definition at the beginning of the section, (w*, p*) satisfies

$$MR_i(w_i^*) = p^*, \quad i = 1, \ldots, n, \quad \text{with} \sum_{i=1}^{n} w_i^* = \frac{n}{2}. \tag{4}$$

Taking p* as the parameter of the RE model, we note that the equilibrium conditions

$$MR_i(w_i^*) = p^*, \quad i = 1, \ldots, n, \tag{5}$$

are the first-order conditions associated with the maximum of league profits Π:

$$\Pi = \sum_{i=1}^{n} \left[R_i(w_i^*) - p^* w_i\right]. \tag{6}$$

We show in Appendix A that the RE equilibrium (w*, p*) exists and is unique, summarized as follows:

Proposition 1: In an NFL-type league, where all teams have strictly concave revenue functions, without revenue sharing, there exists the unique RE equilibrium (w*, p*) from expression (4).

In this simplest case, it ends up that information about other teams is irrelevant to any given team and the RE characterization in equation (3) is not very exciting in terms of information content. One could characterize this as a 'trivial' RE outcome but it does set the stage for the more interesting cases that follow.

Consider now an NFL-type league with shared revenues. The sharing mechanism we employ applies to all revenues R_i. All teams contribute a share of revenues to a pool that is then shared equally by all teams. Since the mid-1990s, this type of pooled revenue sharing has characterized both the NFL and MLB. Kesenne (2005) employs the same device and notes (p. 101) that this type of sharing characterizes some European leagues and UEFA's Champions League. We stress that using this type of sharing technically weakens comparison with past results (like Fort and Quirk, 1995; Kesenne, 1996, 2000; Marburger, 1997; Szymanski, 2004) since those papers covered gate revenue sharing only.

Let $\alpha > \frac{1}{2}$ be the portion of revenues kept by each team so that $1 - \alpha$ is contributed to the sharing pool by each team. Profits for team i become

$$\pi_i = \alpha R_i(w_i) + \left(\frac{1-\alpha}{n}\right) \sum_{j=1}^{n} R_j - p w_i, \quad i = 1, \ldots, n. \tag{7}$$

When $\alpha = 1$, we again get equation (1). First-order conditions for a maximum of profits in equation (7) are

$$\frac{d\pi_i}{dw_i} = \alpha MR_i + \left(\frac{1-\alpha}{n}\right) \sum_{j=1}^{n} MR_j \frac{dw_j}{dw_i} - p = 0, \quad i = 1, \ldots, n, \tag{8}$$

where $\frac{dw_j}{dw_i}$ is team i's expectation of the change in w_j induced by a 1 unit increase in w_i.

We have the usual demonstration of the Rottenberg (1956) invariance principle for $n = 2$ where the adding-up constraint becomes $w_1 + w_2 = 1$ and $\frac{dw_1}{dw_2} = \frac{dw_2}{dw_1} = -1$. In the two-team league, expression (8) yields:

$$\begin{aligned}\alpha MR_1 + \left(\frac{1-\alpha}{2}\right) MR_1 - \left(\frac{1-\alpha}{2}\right) MR_2 \\ = \alpha MR_2 + \left(\frac{1-\alpha}{2}\right) MR_2 - \left(\frac{1-\alpha}{2}\right) MR_1,\end{aligned} \tag{9}$$

so that $MR_1 = MR_2$ and, compared with the case of no revenue sharing, the distribution of winning percents is invariant with respect to imposing pooled revenue sharing. The fact that this holds for arbitrary $n > 2$ is clear from the following.

In this pooled revenue-sharing case, for an NFL-type league, RE equilibrium occurs for a pair (p^{**}, w^{**}) such that

$$\alpha MR_1(w_i^{**}) + \left(\frac{1-\alpha}{n}\right) \sum_{j=1}^{n} MR_j(w_j^{**}) \frac{dw_j}{dw_i} = p^{**}, \quad i = 1, \ldots, n,$$

and

$$\sum_{i=1}^{n} w_i^{**} = \frac{n}{2}. \tag{10}$$

For uniqueness, we add a 'dominant direct effects' (DDE) assumption. In a change from one feasible allocation w' to another w'', an increase in w_i in moving from w' to w'' implies that MR_i (including any induced effects) is less in w'' than in w'. In equation (8), let $MR_i = \alpha MR_i \left(\frac{1-\alpha}{n}\right) \sum_{j=1}^{n} MR_j \frac{dw_j}{dw_i}$. We state the following:

DDE assumption

Given w' and w'' such that $\sum_{i=1}^{n} w'_i = \sum_{i=1}^{n} w''_i$, $w''_i > w'_i$ implies that $MR''_i < MR'_i$, i.e., the direct effect on MR_i of the increase in w_i dominates any induced effects on MR_i.

This is a global condition. Locally, the counterpart to the DDE assumption is the assumption that, evaluated at an RE equilibrium, the Jacobian matrix of the system is a dominant diagonal.

Under the DDE assumption, in the case of NFL-type leagues with pooled revenue sharing, the proof that the RE equilibrium exists and is unique is in Appendix A. We state the global result as follows.

Proposition 2: In an NFL-type league where all teams have strictly concave revenue functions, with pooled revenue sharing, under the DDE assumption, there exits the unique RE equilibrium (w**, p**) such that $p^{**} = \alpha p^*$ and $w^{**} = w^*$ in Proposition 1.

In this revenue-sharing case, RE elements are decidedly more interesting. This is clear in the details of the indirect effects in equation (8), i.e., $\left(\frac{1-\alpha}{n}\right)\sum_{j\neq i}^{n} MR_j \frac{dw_j}{dw_i}$. Each team must take into account the impact of its talent choice on the winning percent outcomes for all other teams, and track that impact through the marginal revenue functions of all other teams that contribute to the shared revenue pool. External effects are internalized because all teams have the information required to calculate these impacts and all teams know that they need to include these impacts as they all come to face the same price, p^{**}. Note that it does not matter to team i how the rest of the teams in the league share the impact of quality choice on their other winning percents, so long as they all also confront the same price of talent; it is the sum of those impacts that matter. But each team has to know that all other teams are maximizing profits *in the same way* for equilibrium to occur.

We finish our examination of the NFL-type league by examining the implications of the RE model as far as the invariance principle is concerned. Invariance in the distribution of talent with respect to the introduction of pooled revenue sharing would be evidence supporting the RE idea for the NFL. Pooled sharing replaced the previous '60–40' gate sharing arrangement in the NFL in 2001. Since then, instead of sharing gate revenue on a visitor-by-visitor basis, 40% of all gate revenue goes into a pot that is shared equally by all teams. The well-known Noll–Scully ratio of standard deviations before straight-pool sharing (1996–2000) was 1.54. And the measure was virtually unchanged after the implementation of straight-pool sharing (2001–2005). These data fail to reject the RE approach and it would be interesting to conduct the same test for other leagues where season ticket sales dominate team revenue functions.

IV RE and the MLB-Type League

In our second case, revenue functions are dominated by single-game ticket sales. This sounds more like MLB to us. With the highest average ticket price across the league closer to \$46, an 81-game season ticket price should be around \$3000. Thus, the larger number of games in the MLB season, even at lower ticket prices, puts the fan at much greater monetary risk should their team perform below expectations. At \$3000, and for a disappointing team, resale would be difficult. Hence, baseball owners are forced into more single-game sales than football owners. It follows that the quality of the visitor plays a much larger role in fan purchase decisions.

In addition to the relatively greater expense of getting stuck with a mediocre season ticket for 81 home games, it is now also commonplace for MLB teams to sell single-game tickets at different prices, depending on the quality of the visiting team. Hence, for leagues like MLB with relatively much longer seasons, both home team and visitor quality will enter directly into the revenue function.

In the absence of any revenue sharing, let strictly concave revenues be R^{ij} (w_i, w_j), the revenues earned by team i playing team j at home. The revenues earned over the season by team i will be the sum of all these individual game revenues, that is, $\sum_{j \neq i}^{n} R^{ij}\left(w_i, w_j\right)$. In this case, the profits for team i equal:

$$\pi^i = \sum_{j \neq i}^{n} R^{ij}(w_i, w_j) - pw_i, \quad i = 1, \ldots, n. \tag{11}$$

The first-order conditions are

$$\frac{d\pi_i}{dw_i} = \sum_{j \neq i}^{n} \left(R_i^{ij} + R_j^{ij} \frac{dw_j}{dw_i} \right) - p = 0, \quad i = 1, \ldots, n, \tag{12}$$

where $R_i^{ij} = \frac{\partial R^{ij}}{\partial w_i}$ and $R_j^{ij} = \frac{\partial R^{ij}}{\partial w_j}$.

Note how the RE element comes into play even without revenue sharing for the MLB-type league. The term under the summation in equation (12) is marginal revenue where each team incorporates the impacts of its talent choice on all other teams' revenues. While equation (12) is still the condition that teams choose winning percent (talent) so that marginal revenues (including induced 'indirect effects') equal the price of talent, marginal revenues again include not just direct effects, but 'indirect effects' on their own home revenues tracked through their talent choice impact on all other teams' winning percents. Indirect impacts affect their own home revenues as the quality of the visiting team matters to home fans. And it does not matter to team i how the rest of the teams in the league share the impact of quality choice on their other winning percents, it is the sum of those impacts that matter. Again, each team has to know that all other teams are maximizing eventually confronting the same price in order to reach equilibrium.

Utilizing the RE definition from before, an RE equilibrium in this case is a pair $(\bar{w}, \bar{p})$ such that:

$$\sum_{j \neq i}^{n} \left(R_i^{ij}(\bar{w}_i, \bar{w}_j) + R_j^{ij}(\bar{w}_j, \bar{w}_j) \frac{dw_j}{dw_i} \right) = \bar{p}, \quad i = 1, \ldots, n, \tag{13}$$

with

$$\sum_{i=1}^{n} \bar{w}_i = \frac{n}{2}.$$

Appendix A presents a proof of existence and uniqueness of the RE equilibrium $(\bar{w}, \bar{p})$, under the DDE assumption. The global result is:

Proposition 3: In an MLB-type league, where all teams have strictly concave revenue functions, without revenue sharing, under the DDE assumption, the RE equilibrium $(\bar{w}, \bar{p})$ given in equation (13) exists and is unique.

Adding pooled revenue-sharing complicates the notation, but not the approach we have already used. Under pooled sharing, profits become

$$\pi^i = \alpha \sum_{j \neq i}^{n} R^{ij}(w_i, w_j) + \left(\frac{1-\alpha}{n}\right) \sum_{k=1}^{n} \sum_{j \neq k}^{n} R^{ji}(w_i, w_j) - pw_i, \quad i = 1, \ldots, n. \tag{14}$$

The first-order conditions are

$$\frac{d\pi_i}{dw_i} = \left[\frac{(n-1)\alpha + 1}{n}\right] \sum_{j \neq i}^{n} \left(R_i^{ij} + R_j^{ij} \frac{dw_j}{dw_i}\right) + \left(\frac{1-\alpha}{n}\right) \sum_{k \neq i}^{n} \left(R_i^{ki} + R_k^{ki} \frac{dw_k}{dw_i}\right) - p = 0, \quad i = 1, \ldots, n \tag{15}$$

with

$$\sum_{i=1}^{n} w_i = \frac{n}{2}.$$

Once again, the RE nature of the equilibrium outcome is apparent, adding impacts of the team's talent choice now on host team revenues when that team is the visitor. All of that is tracked back through the impact on the contribution of those other host teams through the pooled sharing arrangement, that is, the term $\left(\frac{1-\alpha}{n}\right) \sum_{k \neq i}^{n} \left(R_i^{ki} + R_k^{ki} \frac{dw_k}{dw_i}\right)$ in expression (15). And as we have noted before, it is the sum of those impacts that matter rather than any particular distribution, just so each team knows that all other teams choose their optimum confronting the same price.

An RE equilibrium in this case is a pair $(\hat{w}, \hat{p})$ such that

$$\left[\frac{(n-1)\alpha + 1}{n}\right] \sum_{j \neq i}^{n} \left(R_i^{ij}(\hat{w}_i, \hat{w}_j) + R_j^{ij}(\hat{w}_i, \hat{w}_j) \frac{dw_j}{dw_i}\right) + \left(\frac{1-\alpha}{n}\right) \sum_{k \neq i}^{n} \left(R_i^{ki}(\hat{w}_i, \hat{w}_k) + R_k^{ki}(\hat{w}_i, \hat{w}_k) \frac{dw_k}{dw_i}\right) = \hat{p}, \quad i = 1, \ldots, n,$$

with

$$\sum_{i=1}^{n} \hat{w}_i = \frac{n}{2}. \tag{16}$$

We have the following proposition in the same way that we generated Proposition 3, where mr_i in our DDE assumption now covers the left-hand side of equation (15) (see the Appendix A).

Proposition 4: In an MLB-type league, where all teams have strictly concave revenue functions, with pooled revenue sharing, under the DDE assumption, the RE equilibrium $(\hat{w}, \hat{p})$ given in equation (16) exists and is unique.

In general, the invariance principle does not hold for an MLB-type league (although it does hold for the case $n = 2$). Among other things, the nature of the externalities present in the case are such that the decentralized decision making does not in general lead to a maximum of league profits.

V Conclusions and Implications

We show that the CMT model used widely in sports economics also generates unique RE equilibrium. This is true in less-complicated leagues where season ticket sales dominate revenue functions and in more-complicated leagues where single-game ticket sales are dominant. This finding requires a 'direct dominance effect' assumption, namely, direct impacts on a team's marginal revenues dominate indirect, induced effects on that team's marginal revenues due to talent adjustments by all of the other teams in the league. While the invariance principle holds for leagues where season ticket sales dominate, it does not generally hold for leagues dependent primarily on season ticket sales. The implications for the CTM model in sports leagues are as follows.

First, RE equilibrium takes care of the theoretical possibility of informational externalities. All participants can calculate the same equilibrium outcome and none will settle for any other choice of talent than the one that maximizes profit given that information. Ignoring the external impacts of their choice would be irrational.

Second, RE moves interpretation of the CTM model away from *how* the equilibrium process needs to function (e.g., Walras' tatonnement as in El Hodiri and Quirk, 1971; Quirk and El Hodiri, 1974) and toward what that process actually produces. And that, of course, is the equilibrium outcome. Under RE, that outcome incorporates all direct and indirect effects of talent choices for all teams in the league.

Third, RE lends theoretical underpinnings to observations about the free flow of information in North American pro sports leagues. In particular, NALs seem completely likely candidates because the information prerequisites for RE exist in the first place. Perhaps amateur leagues and college conferences in North America, and/or pro leagues around the world, are non-cooperative. If that is the case, it is difficult to see how the same type of RE equilibrium could prevail since non-cooperative outcomes inevitably are tied up with some form of asymmetric information or difference in information sets for participants. It would be helpful if future works explain why such an information setting would characterize any league where non-cooperative models are applied.

Finally, empirically testable propositions are suggested that have the potential to reject the RE theory developed here. One is the cursory examination in a preceding section – does the invariance principal hold for the introduction of pooled revenue sharing? Again, it would be interesting to test this on other NFL-type leagues.

The second empirically testable proposition brings us back to the DDE assumption. As mentioned previously, works using diagrams with a downward-sloping marginal revenue of talent functions have implicitly made the DDE assumption. Owners choose the level of talent (team quality) that will maximize profits taking into account indirect effects compared with all other possible quality levels. And the 'demand' for talent is derived from this consideration of alternatives. But there is no reason *a priori* that this collection of considerations must result in a downward-sloping relationship. Hence, unless the DDE assumption actually holds, marginal revenue of talent functions (including induced effects) can slope up.

These findings provide formal justification for the CTM model of sports leagues and the major implication that leagues are organized to maximize league-wide profits. This casts the CTM approach in an updated, modern light relative to other alternative models of the sports league talent market (see Szymanski, 2004), as well as to the rest of the industrial organization literature. Perhaps this RE equilibrium finding offers the strongest grounds for the relevancy of the CTM model in the analysis of sports leagues.

Acknowledgement

We are indebted to Mohammed El Hodiri for his guidance on existence proofs.

Appendix A: Uniqueness and Existence of Rational Expecations Equilibrium

NFL-type league, no revenue sharing

We wish to show that the RE equilibrium (w^*, p^*) in equation (4) is unique and that it exists. Let $w(p)$ denote the team (and league) profit-maximizing choice of w for each p. By strict concavity, $w(p)$ is unique to every p.

Uniqueness: Now, to prove uniqueness of the equilibrium in equation (4), suppose instead there are two RE equilibriums, (w^*, p^*) and (w^{**}, p^{**}). If the two equilibriums are not identical to each other, then for some team i $w_i^* > w_i^{**}$ and for another team k $w_k^{**} > w_k^*$. If so, then both $MR_i(w_i^{**}) > MR_i(w_i^*)$ and $MR_k(w_k^{**}) < MR_k(w_k^*)$ would hold simultaneously. But by expression (4), this means that both $p^* < p^{**}$ and $p^* > p^{**}$. Proof by contradiction establishes the uniqueness of the (w^*, p^*) equilibrium in equation (4). Uniqueness is cheap for this NFL-type league.

Existence: Team profit-maximizing first-order conditions generate the same (w, p) combination as maximizing league profits subject to the adding-up constraint. Existence of a solution to the *league*-maximization problem follows from the Weierstrass theorem; as the adding-up constraint makes for a compact set, then a continuous function defined on that set has a maximum for that set. As the league and team problems yield the same result, uniqueness is established. Let $H = \sum_i \pi_i$ and let $S = \left\{(w,p) | 0 \le w_i \le 1; \sum_i w_i = \frac{n}{2}; 0 \le p \le \max_i MR_i(w_i)\right\}$. Then H is a continuous function defined over a compact set and the Weierstrass Theorem applies. At an interior maximum, $H_i(w^*, p^*) = MR_i\left(w_i^*\right) - p^* = 0$ and $R_i(w^i)$ is strictly concave by assumption. Hence, (w^*, p^*) exists as an RE equilibrium.

NFL-type league, with revenue sharing

We wish to show that the RE equilibrium (w^{**}, p^{**}) in equation (10) is unique and it exists. Note that $w^{**} = w^*$ and $p^{**} = \alpha p^*$ satisfies equation (10), that is, $\alpha p^* + (1-\alpha)p^* \sum_j \frac{dw_j}{dw_i} = \alpha p^* = p^{**}$ as $\sum_j \frac{dw_j}{dw_i} = 0$.

Uniqueness: Given that mr_i is strictly concave for every i, at an RE equilibrium (w', p'), mr_i is unique for any p. Let (w′, p′) and (w″, p″) be two RE equilibria, so that $mr_i(w') = p'$ for every i, $mr_i(w'') = p''$ for every i, and $\sum_{i=1}^{n} w'_i = \sum_{i=1}^{n} w''_i = \frac{n}{2}$. Assume that $w' \neq w''$. Then, for some k, j, $w'_k > w''_k$ and $w'_j > w''_j$. By the DDE assumption, $mr_k(w') < mr_k(w'')$ and $mr_j(w'') < mr_j(w')$. On the other hand, $mr_k(w') = mr_j(w') = p'$, and $mr_k(w'') = mr_j(w'') = p''$, by the RE equilibrium conditions, so that we have $p' < p''$ and $p'' < p'$. The contradiction establishes uniqueness.

Existence: Let $H = \alpha \sum_i \pi_i$ with S as specified earlier in Appendix A. $H_i(w^*, p^*) = \alpha MR_i(w_i^*) - \alpha p^* = 0$ implies $w_i = w_i^*$ and strict concavity of R_i implies a maximum for H. Hence, again, the Weierstrass theorem applies and the RE equilibrium exists.

MLB-type league, no revenue sharing

We wish to show that the RE equilibrium $(\bar{w}, \bar{p})$ in equation (13) is unique and it exists. From equation (12), our DDE assumption makes $mr_i = \sum_{j \neq i}^{n} \left(R_i^{ij} + R_j^{ij} \frac{dw_j}{dw_i} \right) > 0$. Again, as earlier, we take $\frac{dw_j}{dw_i} = -\frac{1}{n-1}$, $i \neq j$, $i, j = 1, \ldots, n$.

Uniqueness: The argument given for uniqueness for the NFL case above applies also to the MLB case, under the DDE assumption, both with and without revenue sharing, so long as the relevant mr_i functions are strictly concave.

Existence: Let $H = \sum_i \int \left(\frac{d\pi_i}{dw_i} \right) dw_i$ so that $H_i = \frac{d\pi_i}{dw_i} = 0 \Rightarrow w = \bar{w}, p = \bar{p}$. By assumption, $\frac{d^2\pi_i}{dw_i^2} < 0$ so that $\int \left(\frac{d_{i\pi}}{dw_i} \right) dw_i$ is concave and the Weierstrass theorem applies so that $(\bar{w}, \bar{p})$ exists as an RE equilibrium.

MLB-type league, with revenue sharing

While the mr_i functions are more complicated in this case than in the case above, clearly the same formal arguments apply, given strict concavity and the DDE assumption to establish uniqueness, and using the H function given above to establish existence.

References

Canes, M. (1974). The social benefits of restrictions on team quality. In R. G. Noll (ed.), *Government and the Sports Business*. Washington, D.C.: Brookings Instituiton, pp. 81–114.

El-Hodiri, M. and Quirk, J. (1971). An economic model of a professional sports league. *Journal of Political Economy*, **70**, pp. 1302–19.

Fort, R. (2000). Antitrust in pro sports. In P. D. Staudohar (ed.), *Diamond Mines: Baseball and Labor*. Syracuse, NY: Syracuse University Press, pp. 109–21.

Fort, R. (2007). Antitrust in the sports industry. In V. J. Tremblay and C. H. Tremblay (eds), *Industry and Firm Studies*. Armonk, NY: M.E. Sharpe Inc., pp. 245–66.

FORT, R. and QUIRK, J. 1995. Cross-subsidization, incentives, and outcomes in professional team sports leagues. *Journal of Economic Literature*, **XXXIII**, pp. 1265–99.

HOLT, C. A. 2006. The Y2K bibliography of experimental economics and social science. Available at http://www.people.virginia.edu/~cah2k/y2k.htm.

KESENNE, S. (1996). League management in professional team sports with win maximizing clubs. *European Journal for Sport Management*, **2**, pp. 14–22.

KESENNE, S. (2000). Revenue sharing and competitive balance in professional team sports. *Journal of Sports Economics*, **1**, pp. 56–65.

KESENNE, S. (2005). Revenue sharing and competitive balance: does the invariance proposition hold? *Journal of Sports Economics*, **5**, pp. 98–106.

MARBURGER, D. R. (1997). Gate revenue sharing and luxury taxes in professional sports. *Contemporary Economic Policy*, **XV**, pp. 114–23.

QUIRK, J. and EL HODIRI, M. (1974). The economic theory of a professional sports league. In R. G. Noll (ed.), *Government and the Sports Business*. Washington, D.C.: Brookings Institution, pp. 33–80.

QUIRK, J. and FORT, R. D. (1999). *Hard Ball: The Uses and Abuses of Market Power in Professional Sports*. Princeton, NJ: Princeton University Press.

ROTTENBERG, S. (1956). The baseball players' labor market. *Journal of Political Economy*, **64**, pp. 242–58.

SZYMANSKI, S. (2004). Professional team sports are only a game: the Walrasian fixed-supply conjecture model, contest-nash equilibrium, and the invariance principle. *Journal of Sports Economics*, **5**, pp. 111–26.

SZYMANSKI, S. and KESENNE, S. (2004). Competitive balance and gate revenue sharing in team sports. *Journal of Industrial Economics*, **52**, pp. 165–77.

VROOMAN, J. (1995). A general theory of professional sports leagues. *Southern Economic Journal*, **61**, pp. 971–90.

Date of receipt of final manuscript: 10 March 2007