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Optimal Competitive Balance in Single-Game Ticket Sports Leagues

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Abstract

The authors extend the theory of optimal competitive balance to leagues where single-game ticket sales dominate revenues. Whether a planner that maximizes the sum of fan and owner surpluses prefers more balance *or less* in such a league depends on the relative magnitude of marginal consumers' surpluses with respect to talent in larger and smaller revenue markets. This relationship is much more complex than previous cases in the literature. Ultimately, then, the determination of whether more balance is preferred to less in any particular sports league requires careful and thorough empirical investigation.

Keywords

Sports Leagues, Competitive Balance, Optimality

Introduction

In this article, we lay out the basic welfare foundation of optimal competitive balance for regular season play in a "closed" sports league dominated by single-game ticket sales. The article extends the Fort and Quirk (in press) approach to simpler season-ticket leagues to a more complex interaction between team owner talent choices in leagues where single-game tickets sales dominate revenues. To us, Major League Baseball (MLB) typifies this setting. Our chosen focus is on

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balance during the regular season and details of all our modeling choices are in the next section of the article. Policy issues are all covered in the previous work and this article seeks only to add one more type of league to the menu deserving consideration.

We address the question of optimal competitive balance comparing the decentralized, profit-maximizing league outcome to the level of competitive balance that maximizes the sum of consumers' and producers' surpluses. We realize that other possible Pareto optimal outcomes might be developed, but the surplus maximizing approach does have the virtue of using theoretical tools that are conceptually amenable to relatively straightforward measurement and comparison in actual leagues. In particular, our main result is that whether the decentralized league result is too much or too little balance, relative to the surplus-maximizing level, is an empirical matter that has received no attention at all. The elements required to actually assess these marginal impacts of talent rearrangements are available data, and careful empirical analysis can settle the issue. Our main point is that the comparison is more complex than in the season-ticket league covered in the previous literature.

The article proceeds as follows. In the section on Optimal Competitive Balance in a Single-Game Ticket League, we compare the distribution of talent in the decentralized league and planner's optimum for a league where revenues are dominated by single-game ticket sales. The section on Empirical Task lays out the details for future empirical assessment. All of our findings suggest that whether more balance is preferred to less rests on empirical questions that have yet to be assessed. Conclusions round out the article in the Conclusions section.

Optimal Competitive Balance in a Single-Game Ticket League

In some leagues, single-game ticket sales dominate team revenue functions. For example, MLB has 81 home games to sell. Team Marketing Report (2008) tabulates average seat prices weighted by the proportions of different types of seats in stadiums for all MLB teams. The highest of these is about $222 for a premium ticket to an LA Dodgers home game (the league average premium ticket is about $76). But even the league average nonpremium ticket price of about $25 suggests a season ticket price over $2,000. The large number of home games in the MLB season puts the fan at substantial monetary risk should their team perform below expectations. If the team is disappointing, resale is tough at season ticket prices over $2,000. So, baseball owners are forced into more single-game sales than owners in leagues with fewer home games to sell, such as National Football League owners (eight home games plus two preseason). Because the fan focus is now game-by-game, the quality of the visitor plays a large role in fan purchase decisions. Indeed, lately, baseball owners have begun variable ticket pricing based on the quality of the opponent. Our first modeling choice is to focus on a single-game ticket league assuming, consistent with the emphasis on a single game, demand depends on own ticket price and both the team's own winning percentage as well as the same for the opposing teams in the league.

For our second modeling choice, the single-game ticket league is analyzed using the "closed league," competitive talent equilibrium model (originally, El Hodiri & Quirk, 1971). Members of a closed league essentially face a completely inelastic supply of talent; "open league" members might increase their talent by importing it from some other league. MLB, in particular, is distinguished on the closed league basis from other world leagues (e.g., world football). Recently, there has been some international talent migration in the other North American Leagues (NALs), but this is not the same thing as an open league. Once the few best international players are in the pool, all teams cannot increase their talent simultaneously.

The competitive talent market distinction is best portrayed by the classical Walrasian tatonnement referee. Using all information on the impacts of one team's talent choice on the other teams in the league, the referee's price comes to rest where no league member would change their talent choice. Although we find this competitive process acceptable to our needs, especially for NALs, we note that the veracity of the competitive talent market choice is currently under contention (see Fort, 2006; Fort & Quirk, 2007; Szymanski, 2004; Szymanski & Kesenne, 2004).

We also model owners as choosing a single price, rather than as price discriminators (our third distinction). As is well known from welfare economics, *perfect* price discrimination would lead the planner and the league to choose the same talent distribution, although the distribution of wealth would likely be different in the two cases. In MLB, suspicions of price discrimination might be fueled by the presence of variable seat prices. (We ignore personal seat licenses [PSLs] because only a few seats are sold this way in MLB and then only at five locations [Arizona, Minnesota, San Diego, San Francisco, and St. Louis].)

We ignore price discrimination for two reasons. First, the degree to which variable seat prices really are a price discrimination mechanism is debatable. Variation by location, quality of opponent, or the team's fortunes as the season progresses, actually sells different fan experiences. Turning to the season ticket sales that do occur in MLB, if selling rights to consecutive season tickets really is an attempt at price discrimination then, just as with magazine subscriptions, we would expect to see some form of inter-temporal "two-for-one deal" marketing approach, but we do not. Instead, a season ticket sells a different product than a single-game ticket, namely, rights to seat location for the entire season (sometimes rolled over successive seasons)—and, depending on the package, other access to food and services. Furthermore, it ends up that any ticket, including season tickets, can be resold in MLB's official "secondary ticket market" created in 2007 (Auchard, 2007). (Indeed, even for those few teams selling them, PSLs can be completely unbundled from the associated season ticket and sold separately, online [www.stadiumpsl.com].) Preclusion of resale is typically thought to be a prerequisite for successful price discrimination. That leaves price discrimination to the small amount of "on-the-spot" wheeling and dealing that can be done by the ticket manager on a day-today-day basis.

The second reason we ignore price discrimination follows from practical observation on owner choices to centralize significant revenue portions at the league level to be redistributed equally among owners. As just noted, ticket resale is centralized through an MLB official reseller. In addition, a portion of television revenues, branded team merchandise, and emerging electronic rights have all been centralized through the league. We join Kahn (2007) in observing that centralizing revenues at the league level limits the degree to which surpluses can be captured.

Our fourth modeling choice is to focus on regular season play. We further assume a league at a given absolute level of play, the major league level, and that all differences among teams at that level are relative differences (extensions are in Kesenne, 2000; Marburger, 1997; Rascher, 1997). This builds Rottenberg's (1956) outcome uncertainty observation into the model because fans care about relative competition.

The remaining modeling choices are as follows. Fifth, we restrict our attention to gate- and attendance-related local revenue that can be portrayed as proportional to ticket price (Heilmann and Wendling, 1976). This abstracts from local TV revenue requiring a caveat in our conclusions section. Sixth, we assume no team-specific contributions to the value of talent (Vrooman, 1996). Seventh, following the observations in Fort and Winfree (2009), the marginal product of talent is assumed constant (constant returns to scale because talent is the long-run choice of team owners) so that characteristics of the underlying contest success function are essentially assumed away.

There is no need to delve into any mechanisms used to alter the league outcome, such as revenue sharing, to derive our comparisons between the league and the planner. The impacts of revenue sharing in rational expectations equilibrium for this type of model are in Fort and Quirk (2007). We simply note that, in the presence of revenue sharing, the comparison ultimately derived from this type of model becomes even more complex.

We adopt the following notation (our assumptions, mostly the same as in Fort and Quirk (in press) are in Appendix A):

$I = \{1, ..., n\}$ is an index of the set of teams in the league.
w_i = winning percentage of team i.
p = market price per unit of winning percentage.
$t_{ij}(w_i, w_j)$ = ticket price for team i against opponent j.
$D_{ij} = D_{ij}(t_{ij}(w_i, w_j), w_i, w_j)$ = demand for tickets for team i against team j.
$\mathrm{MRP}_i = \frac{\mathrm{d}}{\mathrm{d}w_i}\left[\sum_{j \neq i}^{n}\left(t_{ij}\, w_{ij}\right)\right]$ = marginal revenue product of a unit of winning percentage.

The profit function for team i is:

$$\pi_i = \left(\sum_{j \neq i}^{n} t_{ij} D_{ij} \right) - p\, w_i \, , \, i = 1, \ldots, n. \tag{1}$$

Note that, consistent with a single-game ticket league, we assume ticket demand depends on own ticket price, driven by both the team's own winning percentage and the winning percentage of opponents, in addition to the quality of both teams entering the demand function directly.

First-order conditions for profit maximization are (see Appendix A):

$$\sum_{j \neq i}^{n} \left(t_{ij}^* \frac{\partial D_{ij}^*}{\partial w_i} \right) - \left(\frac{1}{n-1} \right) \sum_{j \neq i}^{n} \left(t_{ij}^* \frac{\partial D_{ij}^*}{\partial w_j} \right) - p = 0, \quad i = 1, \ldots, n, \quad \textit{and} \quad \sum_{i=1}^{n} w_i = \frac{n}{2}. \tag{2}$$

Expression (2) makes it clear that each single-game ticket owner's demand for winning percentage (MRP under our Measurement Convention) includes the impact of their talent choice on all of the other demand functions in the league.

Let $w^* = \left(w_1^*, \ldots, w_n^* \right)$ be the profit-maximizing vector of talent in the league. There is a league profit-maximizing equilibrium at the optimal ticket price vector, talent choice vector, and price of talent $(t*, w*, p*)$, if (Appendix A):

$$F_{ij}^* - F_{ki}^* = \left(\frac{1}{n-1} \right) \left(G_{ij}^* - G_{ki}^* \right), \quad j, k = 1, \ldots, n \quad \textit{and} \quad \sum_{i=1}^{n} w_i^* = \frac{n}{2}, \tag{3}$$

where

$$F_{ij}^* = \sum_{j \neq i}^{n} \left(t_{ij}^* \frac{\partial D_{ij}^*}{\partial w_i} \right), \quad F_{ki}^* = \sum_{k \neq i}^{n} \left(t_{ki}^* \frac{\partial D_{ki}^*}{\partial w_k} \right),$$

$$G_{ij}^* = \sum_{j \neq i}^{n} \left(t_{ij}^* \frac{\partial D_{ij}^*}{\partial w_j} \right), \quad G_{ki}^* = \sum_{k \neq i}^{n} \left(t_{ki}^* \frac{\partial D_{ki}^*}{\partial w_i} \right), \quad j, k = 1, \ldots, n.$$

For what follows, we observe two implications from Expression (3). First, given our ticket price assumption, this equilibrium also has total revenue maximized for the league as a whole. We make use of this observation in our specification of the planner's optimum, shortly. Second, decentralized profit maximization will also maximize overall league total revenues because this is rational expectations equilibrium. Finally, in our model, it is possible for league revenues to be maximized, consistent with (3), at a constant vector $w* = 0.500$, that is, a perfectly balanced league. However, Expression (5) makes it clear that this can only occur if marginal revenue products are equal across all teams *and for all values of* w. But as long as there is variation in talent demand, itself, the league cannot be perfectly balanced.

We make headway analyzing league competitive balance at this point by imposing well-ordered variation in ticket demands across the teams in the league:

Globally Invariant Drawing Power (GIDP) Assumption

Assume the set of teams $I = \{1, ..., n\}$ is listed in order of drawing power such that, for $i > j$, $w_i > w_j$ implies:

$D_{ij}(t_{ij}(w_i, w_j), w_i, w_j) > D_{ji}(t_{ji}(w_j, w_i), w_j, w_i)$ for any common $t_{ij}, t_{ji} \geq 0$ and

$D_{ij}^{-1}(q_{Dij}, w_i, w_j) > D_{ji}^{-1}(q_{Dji}, w_j, w_i)$ for any common q_{Dij}, q_{Dji} where q_{Dij} is the number of tickets demanded for team i against team j.

$i > j$ implies $t^*_{ij} > t^*_{ji}$ in equilibrium.

Generally speaking, this is the well-known larger and smaller revenue market distinction common in the analysis of sports leagues. Under the GIDP assumption, team $i < j$ occupies the relatively larger revenue market; Team 1 occupies the largest revenue market; Team 2 is in the next largest revenue market, and so on down to team n. This seems reasonable especially over any relevant team or league planning horizon because the location of teams helps determine their drawing power and team location is completely in the hands of the league itself.

It is straightforward to show that the decentralized league equilibrium exhibits competitive imbalance with larger revenue market teams winning more than smaller revenue market teams (see Appendix A). As discussed previously, the only time this will not be true is if (removing the GIDP assumption) there are no larger and smaller revenue markets to begin with, that is, talent demand is identical in all markets so that equilibrium occurs at 0.500 for all teams.

We next consider the planner's optimum. For simplicity, we have the planner take the monopoly pricing power of each team as given. Let fans' surpluses in market i when team j is the visitor be denoted by $C_{ij} = \int_{t_{ij}}^{\infty} D_{ij}\left(t_{ij}, w_i, w_j\right) d\, t_{ij}$. Team surpluses (revenues) from that same game are $R_{ij} = t_{ij}\, D_{ij}\left(t_{ij}, w_i, w_j\right)$. The planner maximizes the sum of these surpluses across all visitors to each team, and then across all teams, incorporating the constraint directly into the first order conditions. Unlike the members of the league in their decentralized equilibrium, rational expectations lend nothing to the planner's decision. Thus, the planner's problem is to maximize the following with respect to w_i:

$$S = \sum_{i=1}^{n} \sum_{j \neq i}^{n} \left(C_{ij} + R_{ij}\right) + \lambda\left(\sum_{i=1}^{n} w_i - \frac{n}{2}\right), \quad i = 1, \ldots, n. \tag{4}$$

Using the t'_{ij} that will eventually maximize revenues anyway in the planner's final equilibrium, the first-order conditions are (see Appendix B):

$$\sum_{j\neq i}^{n}\int_{t'_{ij}}^{\infty}\frac{\partial D'_{ij}}{\partial w_i}d\,t_{ij}+t'_{ij}\frac{\partial D'_{ij}}{\partial w_i}+\sum_{k\neq i}^{n}\int_{t'_{ki}}^{\infty}\frac{\partial D'_{ki}}{\partial w_i}d\,t_{ki}+t'_{ki}\frac{\partial D'_{ki}}{\partial w_i}=\lambda,$$
$$i=1,\ldots,n, and \sum_{i=1}^{n}w_i=\frac{n}{2}. \quad (5)$$

The Lagrange multiplier λ captures all the induced changes in surpluses caused by the change in any given w_i. Theoretically, this means that the solution for λ would allow solution for all $\frac{dw_j}{dw_i}$. Let $w' = (w'_1, ..., w'_n)$ be the welfare-maximizing talent vector. Then there is a planner's welfare-maximizing equilibrium at the optimal ticket price vector, talent choice vector, and price of talent (t', w', p'), if (see Appendix B):

$$F'_{ij}-F'_{ki}=Y'_{ki}-Y'_{ij}, j,k=1,\ldots,n, and \sum_{i=1}^{n}w_i=\frac{n}{2}, \quad (6)$$

where:

$$F'_{ij}=\sum_{j\neq i}^{n}\left(t'_{ij}\frac{\partial D'_{ij}}{\partial w_i}\right),\ F'_{ki}=\sum_{k\neq i}^{n}\left(t'_{ki}\frac{\partial D'_{ki}}{\partial w_k}\right),$$

$$Y'_{ij}=\sum_{j\neq i}^{n}\left[\int_{t'_{ij}}^{\infty}\frac{\partial D'_{ij}}{\partial w_i}d\,t_{ij}\right],\ Y'_{ki}=\sum_{k\neq i}^{n}\left[\int_{t'_{ki}}^{\infty}\frac{\partial D'_{ki}}{\partial w_i}d\,t_{ki}\right].$$

The left-hand side of the planner's equilibrium in (6) includes the same considerations as the league's profit-maximizing equilibrium in the left-hand side of (3). Of course, these terms may be evaluated at $w' \neq w*$. But the right-hand side of (6) is completely different than the right-hand side of (3). The league only cares about talent impacts on marginal revenue products but the planner has more on their plate. [We note in passing that it is possible for welfare to be maximized, consistent with (6), at a constant vector $w' = 0.500$, that is, a perfectly balanced league but as long as there is variation in talent demand, itself, the league cannot be perfectly balanced.]

The comparison between the planner's optimum and the league profit-maximizing equilibrium reduces to the sign of the following:

$$M=Y'_{ki}-Y'_{ij}-\left(\frac{1}{n-1}\right)\left(G^*_{ij}-G^*_{ki}\right), j,k=1,\ \ldots,n. \quad (7)$$

The intuition from the mathematics is as follows (intuition about the elements of the comparison are the subject of the next section). As a benchmark, the league's profit-maximizing talent distribution is the same as the planner's welfare-maximizing talent distribution if $M = 0$. The first-order conditions in (3) and (6) would be identical in this case and the planner would simply set $w' = w*$. It would follow that the same ticket price vector chosen by the league would maximize revenues, $t' = t*$.

If $M > 0$, so that $Y'_{ki} - Y'_{ij} > \left(\frac{1}{n-1}\right)\left(G^*_{ij} - G^*_{ki}\right) > 0$, j, k = 1, ..., n., then, relative to the case where $w' = w*$ and $t' = t*$, the planner must rearrange talent *away from the larger-revenue owner* and *toward the smaller-revenue owner* in order to make the left-hand side positive because $\frac{\partial MRP_i}{\partial w_i} < 0$ (Talent Demand Assumption). Thus, *more balance* is welfare-enhancing if the right-hand side of (7) is positive. But if $M < 0$, then the planner must rearrange talent toward the larger-revenue owner and less balance is welfare enhancing. The upshot of all of this is that the determination of a league's choice of balance, compared to one that would maximize welfare as defined here, is ultimately a complicated empirical task. The next section is a detailed intuitive discussion of expression (7) as a guide to that task and a comparison to the simpler case of a season-ticket league (Fort and Quirk, forthcoming).

III. Empirical Task

Determining the sign of M in expression (7) answers whether increasing or decreasing the level of balance in a single-game ticket league will be welfare enhancing. By our GIDP Assumption, the discussion proceeds with team i as the largest-revenue market team, j the next largest, and k the relatively smallest-revenue market team. Generally, then, according to (7), the answer depends on the impacts of increasing talent in smaller-revenue markets compared to the other larger-revenue markets. And care must be exercised to estimate these impacts for each opponent since the comparison is eventually decided by the sum of changes in surpluses across all opponents.

Theory alone cannot decide the outcome. While our Attendance Demand Assumption and Ticket Price Assumption dictate that $Y'_{ki} > 0$ and $Y'_{ij} > 0$, $Y'_{ki} - Y'_{ij}$ cannot be signed strictly in our theory. For $Y'_{ki} - Y'_{ij}$, how do incremental impacts on fans (that is, the areas under the new demand curves for each home game) sum up across all home games in a given market? Then, how do those totals compare between markets? These complications are partly related to the results in Davis (2009) where a team like the Yankees may be a good road draw, but not at the same level at two different teams. Fort and Quirk (forthcoming) provide the full discussion of where the rest of the literature is on these empirical issues.

In conclusion, we note that the task just laid out for the single-game ticket league is more complex than in previous work on season-ticket dominated leagues. Fort and Quirk (forthcoming) derive the following analog to (7) for the *season-ticket league*, (their expression (10)). Whether more or less balance is welfare enhancing for the season-ticket league depends on the sign of the following:

$$M_{ST} = \int_{t'_j}^{\infty} \frac{\partial D'_j}{\partial w_j} d\,t_j - \int_{t'_i}^{\infty} \frac{\partial D'_i}{\partial w_i} d\,t_i, i,j = 1, \ldots, n. \quad (8)$$

Again, team i is in the larger-revenue market and team j is in the smaller-revenue market. Further, demand only depends on own team performance for teams in a season-ticket leagues and there is no summation across all home games. So, the difference between the league revenue-maximizing equilibrium result and the planner's optimum for the season-ticket league rests solely on shifts in demand in the smaller-revenue market, compared to the larger-revenue market.

Comparing (8) for the season-ticket league and (7) for the single-game ticket league shows that the former is almost trivial compared to the latter! The terms of interest in the much simpler season-ticket league case involve no summation across all home games. Further, the only comparison across the smaller-revenue market and larger-revenue market are the direct effects on fan surpluses. Pity the planner in the more complex single-game ticket league case!

IV. Conclusions

We extend the analysis of optimal balance to the single-game ticket league (like Major League Baseball). The planner's optimal talent distribution for regular season play maximizes the sum of fans' and owners' surpluses. That outcome is compared to the decentralized, profit-maximizing outcome for the same league. We also compare our results to the previous results in the literature for season-ticket leagues (like the National Football League).

As long as owners in different locations face different attendance demand functions, the profit-maximizing outcome for single-game ticket leagues is competitive imbalance. Whether such a single-game ticket league has too little or too much balance, relative to the maximization of surpluses, depends on the impacts of increasing talent in smaller-revenue markets compared to the other larger-revenue markets. In addition, the empirical task for the single-game ticket league is much more complex than for season-ticket league treated previously in the literature. Suggestions for future research are all covered already in Fort and Quirk (forthcoming). Our intention here is simply to extend the logic of optimal balance to the single-game ticket league.

Appendix A: The Profit-Maximizing Equilibrium

And our assumptions are as follows:

Measurement Assumption

Suppose z_i is talent used by team i to produce its own winning percentage. We assume that z_i is measured so that the marginal product of talent is one, that is, $\frac{\partial w_i}{\partial z_i} = 1$. This assumption has implications. First, the price of a unit of winning percentage, p, is also the price of a unit of talent. Second, the marginal revenue product of winning percentage, MRP_i, is also the marginal revenue product of talent, that is, the demand for talent. Limitations imposed by this assumption for this model are explored in detail in Fort and Winfree (2009).

Ticket Price Assumption

$\frac{\partial t_{ij}}{\partial w_i} > 0$, fans are willing to pay more for higher own-team quality measured by winning percentage. Furthermore, for any choice of (w_i, w_j), t_{ij} is chosen to maximize revenue for that (w_i, w_j). For a general concave revenue function, $\sum_{j \neq i}^{n} t_{ij} D_{ij}$, let t_{ij}^* distinguish this fact and let $D_{ij}^* = D_{ij}\left(t_{ij}^*\left(w_i, w_j\right), w_i, w_j\right)$. The implication of revenue maximization with respect to t_{ij} is that marginal revenues are 0, that is, $t_{ij}^* \frac{\partial D_{ij}^*}{\partial t_{ij}} + D_{ij}^* = 0$, further implying the elasticity of ticket demand *with respect to ticket price* will always be unity, that is, $-\left(\frac{t_{ij}^*}{D_{ij}^*} \frac{\partial D_{ij}^*}{\partial t_{ij}}\right) = 1$. Note that this is *not* the same thing as maximizing revenue with respect to w_i.

Attendance Demand Assumptions

$\frac{\partial D_{ij}}{\partial t_{ij}}$, so that ticket demand slopes downward; $\frac{\partial D_{ij}}{\partial w_i} > 0$ with $\frac{\partial^2 D_{ij}}{\partial w_i^2}$ so that increased quality shifts demand to the right, but at a decreasing rate.

Talent Demand Assumption

$\frac{\partial MRP_i}{\partial w_i} < 0$, the demand for talent slopes downward.

We also make use of the following additional implications of the "closed league" characterization that are not used by Fort and Quirk (in press) for reasons soon to be clear.

Close League Implication

For a closed league, under the adding-up constraint, if team i increases its talent, the remaining owners must decrease theirs by the same amount, that is, $\sum_{j \neq i}^{n} \frac{d w_j}{d w_i} = -1$.

Rational Expectations Implication

This specific closed-league model also has rational expectations equilibrium (Fort & Quirk, 2007). All participants know that eventually they will face the same market price of talent, p, so they will all adjust their talent choices, identically. In a closed league, this produces the following implication:

$$\sum_{j\neq 1}^{n}\frac{dw_j}{dw_i}=-1\Rightarrow\frac{dw_j}{dw_i}=\left(-\frac{1}{n-1}\right),\quad j\neq 1.$$

From the profit function in (1), first-order conditions are:

$$\frac{d\pi_i}{dw_i}=\sum_{j\neq i}^{n}\left(\frac{\partial t_{ij}}{\partial w_i}+\frac{\partial t_{ij}}{\partial w_j}\frac{d\,w_j}{d\,w_i}\right)D_{ij}$$
$$+\sum_{j\neq i}^{n}\left[\frac{\partial D_{ij}}{\partial t_{ij}}\left(\frac{\partial t_{ij}}{\partial w_i}+\frac{\partial t_{ij}}{\partial w_j}\frac{d\,w_j}{d\,w_i}\right)+\frac{\partial D_{ij}}{\partial w_i}+\frac{\partial D_{ij}}{\partial w_j}\frac{d\,w_j}{d\,w_i}\right]t_{ij}-p=0, \tag{A1}$$
$$i=1,\ldots,n,\quad \textit{and}\quad \sum_{i=1}^{n}w_i=\frac{n}{2}.$$

Our rational expectations implication is that $\frac{d\,w_j}{d\,w_i}=\left(-\frac{1}{n-1}\right),\quad j\neq i$. Invoking this implication in no way destroys comparability to Fort and Quirk (in press), because the season-ticket league model has no complication involving these terms at all; they could have invoked this implication if it had mattered at all in their league first-order conditions. Under this implication, (A1) reduces to:

$$\sum_{j\neq i}^{n}\left[\frac{\partial t_{ij}}{\partial w_i}D_{ij}+t_{ij}\left(\frac{\partial D_{ij}}{\partial t_{ij}}\frac{\partial t_{ij}}{\partial w_i}+\frac{\partial D_{ij}}{\partial w_i}\right)\right]$$
$$-\left(\frac{1}{n-1}\right)\sum_{j\neq i}^{n}\left[\frac{\partial t_{ij}}{\partial w_j}D_{ij}+t_{ij}\left(\frac{\partial D_{ij}}{\partial t_{ij}}\frac{\partial t_{ij}}{\partial w_j}+\frac{\partial D_{ij}}{\partial w_j}\right)\right]-p=0, \tag{A2}$$
$$i=1,\ldots,n,\quad \textit{and}\quad \sum_{i=1}^{n}w_i=\frac{n}{2}.$$

Furthermore, remembering that ticket price always adjusts to maximize revenue for any (w_i,w_j), our ticket price assumption implies that $t^*_{ij}\frac{\partial D^*_{ij}}{\partial t_{ij}}=-D^*_{ij}$ so that (A2) becomes (Expression (2) in the text of the article):

$$\sum_{j\neq i}^{n}\left(t^*_{ij}\frac{\partial D^*_{ij}}{\partial w_i}\right)-\left(\frac{1}{n-1}\right)\sum_{j\neq i}^{n}\left(t^*_{ij}\frac{\partial D^*_{ij}}{\partial w_j}\right)-p=0,\ \ i=1,\ldots,n,\ \ \textit{and}\ \ \sum_{i=1}^{n}w_i=\frac{n}{2}. \tag{A3}$$

Let $w^*=\left(w^*_1,\ldots,w^*_n\right)$ be the profit-maximizing vector of talent in the league. Thus, in equilibrium (A3) implies there is a league profit-maximizing equilibrium at the optimal ticket price vector, talent choice vector, and price of talent $(t*,w*,p*)$, if:

$$\sum_{j\neq i}^{n}\left(t_{ij}^{*}\frac{\partial D_{ij}^{*}}{\partial w_i}\right)-\left(\frac{1}{n-1}\right)\sum_{j\neq i}^{n}\left(t_{ij}^{*}\frac{\partial D_{ij}^{*}}{\partial w_j}\right)$$
$$=\sum_{k\neq i}^{n}\left(t_{ki}^{*}\frac{\partial D_{ki}^{*}}{\partial w_i}\right)-\left(\frac{1}{n-1}\right)\sum_{k\neq i}^{n}\left(t_{ki}^{*}\frac{\partial D_{ki}^{*}}{\partial w_i}\right),\ j,k=1,\ldots,n,\quad and\quad \sum_{i=1}^{n}w_i^{*}=\frac{n}{2}. \tag{A4}$$

For comparison purposes later, we choose to write (A4) as (Expression (3) in the text of the article):

$$F_{ij}^{*}-F_{ki}^{*}=\left(\frac{1}{n-1}\right)\left(G_{ij}^{*}-G_{ki}^{*}\right),\quad j,k=1,\ldots,n,\quad and\quad \sum_{i=1}^{n}w_i^{*}=\frac{n}{2}, \tag{A5}$$

where

$$F_{ij}^{*}=\sum_{j\neq i}^{n}\left(t_{ij}^{*}\frac{\partial D_{ij}^{*}}{\partial w_i}\right),\quad F_{ki}^{*}=\sum_{k\neq i}^{n}\left(t_{ki}^{*}\frac{\partial D_{ki}^{*}}{\partial w_k}\right),$$

$$G_{ij}^{*}=\sum_{j\neq i}^{n}\left(t_{ij}^{*}\frac{\partial D_{ij}^{*}}{\partial w_j}\right),\quad G_{ki}^{*}=\sum_{k\neq i}^{n}\left(t_{ki}^{*}\frac{\partial D_{ki}^{*}}{\partial w_i}\right),\quad j,k=1,\ldots,n.$$

In an n-team single-game ticket league, with a competitive talent equilibrium, with globally invariant drawing power among teams, league revenues are maximized at ticket price and winning percentage vectors (t*, w*) satisfying (7) if and only if $t_{ij}^{*}>t_{jk}^{*}$ and $w_i^{*}>w_j^{*}$ for all$j\neq i$ and $k\neq j$.

We have $t_{ij}^{*}>t_{jk}^{*}$ as part of the GIDP assumption. To see that $w_i^{*}>w_j^{*}$, for all $j\neq i$ and $k\neq j$, note that $\sum_{j\neq i}^{n}t_{ij}^{*}\left[\frac{\partial D_{ij}^{*}}{\partial w_i}+\frac{\partial D_{ij}^{*}}{\partial w_j}\left(-\frac{1}{n-1}\right)\right]=MRP_i^{*}$ in (5). The GIDP assumption has $\text{MRP}_i > \text{MRP}_j$ for any $0\leq w\leq 1$ and for all $j\neq i$. In turn, at the particular p* satisfying (5), $p*=MRP_i^{*}=MRP_j^{*}$ implies $w_i^{*}>w_j^{*}$ for all $j\neq i$.

Appendix B: The Planner's Optimum

Looking forward to the fact that the planner takes as given that ticket prices adjust optimally, we start from the definition of consumers' surpluses and producers' surpluses, respectively:

$$C_{ij}=\int_{t_{ij}}^{\infty}D_{ij}\left(t_{ij},w_i,w_j\right)d\,t_{ij}, \tag{B1}$$

$$R_{ij}=t_{ij}\,D_{ij}\left(t_{ij},w_i,w_j\right). \tag{B2}$$

The Lagrangian is (Expression (4) in the text of the article):

$$S = \sum_{i=1}^{n} \sum_{j \neq i}^{n} \left(C_{ij} + R_{ij} \right) + \lambda \left[\sum_{i=1}^{n} w_i - \frac{n}{2} \right], \; i = 1, \ldots, n. \tag{B3}$$

In the planner's problem, each first-order condition is explicitly $\frac{\partial S}{\partial w_i}$, with all w_j held fixed, $j \neq i$. Thus, no terms involving $\frac{d w_j}{d w_i}$, $j \neq i$, will appear in the planner' first-order conditions—the impact on total surplus due to the induced changes in w_j because a change in w_i must be captured by the Lagrange multiplier. First-order conditions for surplus maximization are:

$$\frac{\partial S}{\partial w_i} = \frac{d}{d w_i} \left[\sum_{i=1}^{n} \sum_{j \neq i}^{n} \left(C_{ij} \right) \right] + \frac{d}{d w_i} \left[\sum_{i=1}^{n} \sum_{j \neq i}^{n} \left(R_{ij} \right) \right] = \lambda, i = 1, \ldots, n, \quad \text{and} \sum_{i=1}^{n} w_i = \frac{n}{2}. \tag{B4}$$

Using the t'_{ij} that will eventually maximize revenues anyway in the planner's final equilibrium, and Leibniz Rule for the derivate of C with respect to w_i, the first-order conditions are written (Expression (5) in the text):

$$\sum_{j \neq i}^{n} \int_{t'_{ij}}^{\infty} \frac{\partial D'_{ij}}{\partial w_i} d\, t_{ij} + t'_{ij} \frac{\partial D'_{ij}}{\partial w_i} + \sum_{k \neq i}^{n} \int_{t'_{ki}}^{\infty} \frac{\partial D'_{ki}}{\partial w_i} d\, t_{ki} + t'_{ki} \frac{\partial D'_{ki}}{\partial w_i} = \lambda, \quad i = 1, \ldots, n, \text{ and} \sum_{i=1}^{n} w_i = \frac{n}{2}. \tag{B5}$$

Note that the usual term evaluated at the upper limit of integration in Leibniz Rule is zero since quantity demanded at an infinite price is zero, that is, $D_{ij}\big|_{\infty} = D_{ki}\big|_{\infty} \to 0$ for all j, k. Further, no terms involving $\frac{\partial t_{ij}}{\partial w_i}$ appear since revenue maximization is subsumed.

Let $w' = (w'_1, \ldots, w'_n)$ be the welfare-maximizing talent vector. Then (B5) implies there is a planner's welfare-maximizing equilibrium at the optimal ticket price vector, talent choice vector, and price of talent (t', w', p'), if:

$$\sum_{j \neq i}^{n} \left[\int_{t'_{ij}}^{\infty} \left(\frac{\partial D'_{ij}}{\partial w_i} \right) d\, t_{ij} + t'_{ij} \frac{\partial D'_{ij}}{\partial w_i} \right] = \sum_{k \neq i}^{n} \left[\int_{t'_{ki}}^{\infty} \left(\frac{\partial D'_{ki}}{\partial w_i} \right) d\, t_{ki} + t'_{ki} \frac{\partial D'_{ki}}{\partial w_i} \right], \quad i = 1, \ldots, n, \text{and} \sum_{i=1}^{n} w_i = \frac{n}{2}. \tag{B6}$$

Using the same convention as for the profit-maximizing league equilibrium, we write (B6) as (Expression (6) in the text of the paper):

$$F'_{ij} - F'_{ki} = Y'_{ki} - Y'_{ij}, j, k = 1, \ldots, n, and \ldots \sum_{i=1}^{n} w_i = \frac{n}{2}, \tag{B7}$$

where:

$$F'_{ij} = \sum_{j \neq i}^{n} \left(t'_{ij} \frac{\partial D'_{ij}}{\partial w_i} \right), \; F'_{ki} = \sum_{k \neq i}^{n} \left(t_{ki} \frac{\partial D'_{ki}}{\partial w_k} \right),$$

$$Y'_{ij} = \sum_{j \neq i}^{n} \left[\int_{t'_{ij}}^{\infty} \frac{\partial D'_{ij}}{\partial w_i} d\, t_{ij} \right], \quad Y'_{ki} = \sum_{k \neq i}^{n} \left[\int_{t'_{ki}}^{\infty} \frac{\partial D'_{ki}}{\partial w_i} d\, t_{ki} \right].$$

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