Normal Backwardation and the Inventory Effect

Rodney Fort

Washington State University

James Quirk

California Institute of Technology

The existence of backwardation in futures markets has remained an intriguing and controversial issue since Keynes first argued that it was the "normal" state of affairs. Most theoretical explanations for the existence of backwardation are quite restrictive. Among the assumptions are pure forward as opposed to true futures trading, differences in probability beliefs, degrees of risk aversion, or the level of commodity commitments among long and short hedgers. In a simple model of short and long commodity hedgers, we show that a backwardation equilibrium can occur in a true futures (as opposed to forward) market even when hedgers are identical in these respects and speculators hold the same probability beliefs as hedgers. This result is driven by Houthakker's completely neglected intuitive notion that one possible explanation for backwardation is the high correlation between cash and futures prices when inventories of a commodity are large. While the simple existence of such an "inventory effect" does not necessarily imply backwardation, we prove that backwardation will occur under an appropriately specified inventory effect.

I. Introduction

The behavior of participants in commodity futures markets is inexorably tied to the relationship between current and expected futures

This work was financed in part under a grant from the National Science Foundation (SES-8319960). We thank Jeff Dubin, David Grether, Donald Lien, Roger Noll, and anonymous referees, all of whom helped make the work better than it would have been otherwise.

[*Journal of Political Economy*, 1988, vol. 96, no. 1]

prices, which is in turn bound to the cash price (spot or forward) by the need to carry stocks through time. Of particular interest in this regard are conditions under which the amount of futures that short hedgers wish to sell will exceed the amount that long hedgers wish to buy; that is, hedging is net short, with the expected value of the futures price exceeding its current quotation (an expected rise in the futures price), or what has become the standard use of that venerable old term, "backwardation."[1]

The purpose of this paper is to derive theoretically sufficient conditions for backwardation under a framework with the following characteristics not generally found in the literature. First, ours is an explicit treatment of a true futures market as opposed to a forward market. Anderson and Danthine (1983) (see also Feder, Just, and Schmitz 1980) provide a detailed treatment of long and short hedging in forward markets, where "perfect" hedges occur, with the emphasis being placed on the kinds of uncertainties faced by different traders and on the limited range of forward markets available to use in trading away such uncertainties. In contrast, the results of the present paper hinge crucially on the fact that true commodity futures markets provide only "imperfect" hedges because a range of delivery alternatives are available to the seller of a futures contract. Our explanation for backwardation rests on well-known institutional features of futures contracts. Second, our approach is in contrast to works that rely on informational asymmetries (Danthine 1978), differences in attitudes toward risk, or essentially ad hoc limitations such as hedging by short hedgers only (Danthine 1978; Baesel and Grant 1982) in order to explain the existence of backwardation. It also provides an alternative to the argument that backwardation can result when futures contracts provide poor consumption hedges (Richard and Sundaresan 1981).

Backwardation is an important topic since its presence would ensure long-run speculative profits. The relationship between backward-

[1] The term "backwardation" has been used in a variety of ways relative to expected and current spot, forward, and futures prices. Popular use of the old trading references, backwardation and contango, in the theoretical literature began with Keynes (1930). In his well-known development of the theory of the risk premium, he repeatedly made use of the original definitions of backwardation, a situation in which the current *spot* price exceeds the current *forward* price, and contango, the reverse situation. One frequent use of the term backwardation comes from Keynes's reference to an excess of the *expected* spot price over the *current forward* price under "normal" supply conditions. While he named this excess the "risk premium," it has subsequently come to be called Keynesian "normal backwardation." This use of the term backwardation has been carried into the analysis of *futures* markets as opposed to the markets Keynes analyzed, *forward* markets. As in this paper, the term is currently used to describe a situation in which the current *futures* price is a downward-biased estimate of its own value at the maturity of the contract.

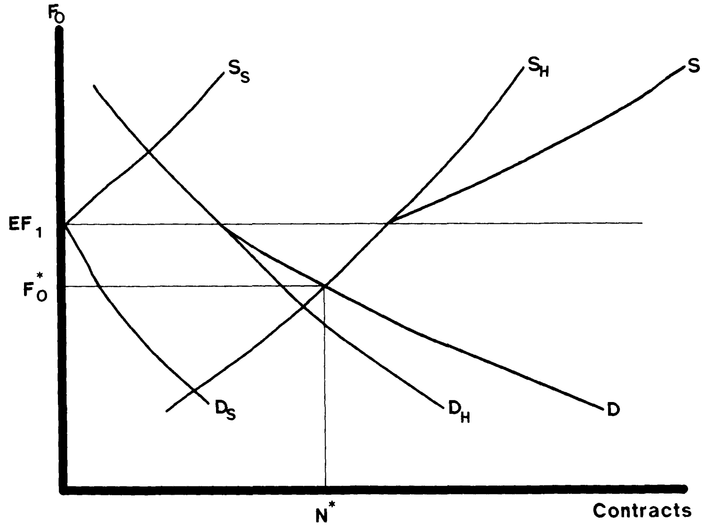

FIG. 1.—A backwardation equilibrium

ation and speculative profits can be neatly summarized with the aid of figure 1. Let F_0 and F_1 be the current and later futures prices, respectively. Speculators (those participants with absolutely no interest in the physical cash commodity, per se, and hedgers who are involved in futures beyond the requirements of their cash position) will demand futures contracts when the futures price is expected to rise ($EF_1 > F_0$). However, speculators supply contracts when the price is expected to fall ($EF_1 < F_0$). This is reflected in the speculative demand curve, D_S, and speculative supply curve, S_S. There also is a demand for futures contracts for long hedging purposes, D_H, and a supply of contracts for short hedging purposes, S_H.

A backwardation equilibrium results when the market-clearing futures price is below the expected later value of the contract. This is labeled (N^*, F_0^*) in figure 1. Should such an equilibrium characterize the futures market, speculators earn long-run profits by simply buying futures contracts. The requirements for such an equilibrium are net short hedging ($S_H > D_H$) at $F_0 = EF_1$ and speculative demand of less than perfect elasticity. This paper examines arguments concerning the former (the latter requirement receives impeccable treatment by Cootner [1960]). In particular, the focus will be on sufficient conditions for net short hedging and backwardation derived from an idea first offered by Houthakker (1957, 1968).[2]

[2] From a historical perspective relevant to the developments by Houthakker, two views can be isolated regarding the behavior of commodity futures prices. One view is

One of Houthakker's arguments for net short hedging rests on the notion that the correlation between cash and futures prices depends on the stocks of the commodity.[3] When inventories of a commodity are large, cash and futures prices for the commodity tend to be highly correlated, while at low inventory levels, cash and futures prices are less highly correlated. We refer to price behavior that depends on the stock cycle in commodities as the "inventory effect."

A high correlation between cash and futures prices tends to encourage both long and short hedging because it makes the futures contract a more effective hedging instrument. Large inventories tend to occur just after the harvest. Houthakker noted that typically the basis (futures price minus cash price) narrows from the harvesttime until several months later. This favors short hedgers, who have bought cash and sold futures, and works to the disadvantage of long hedgers. Hence, during a period from harvest until several months later, short hedging tends to dominate long hedging. Again, typically, from near the middle of the crop year until near the harvest, the basis widens. This benefits long hedgers at the expense of short hedgers, but by this time inventories have become somewhat depleted, which weakens the correlation between cash and futures prices and acts to discourage both long and short hedging. Thus Houthakker arrived at an argument for backwardation as a seasonal phenomenon occurring during the period when inventories are large and the basis is narrow-

that there is no trend in futures prices since expectations are brought to bear equally on all commodity prices, cash (spot and forward) and futures (Hawtrey 1940; Working 1948, 1949, 1953*a*, 1953*b*; Telser 1958, 1960, 1967). This is due to the connection over time provided by carryover stocks from harvest to harvest. Other works tending to support this view include Gray (1961), Rockwell (1967), and Dusak (1973). Another view admits the possibility of seasonal trends in the futures price, based on the behavior of hedgers relative to stock levels over the harvest cycle (Houthakker 1957, 1968; Brennan 1958; Cootner 1960, 1967). These two basic views spawned the well-known debate in the literature on futures markets over backwardation and the existence of long-run speculative profits from a simple strategy of buying futures. The two views stem from much early work on futures markets. An early point of theoretical contention was whether or not futures prices should rise throughout the duration of the contract, especially when there are stocks in excess of the level required to maintain production at normal levels. In a futures market (see Keynes [1930] and Hicks [1946] for forward markets), Dow (1940) argued that commodity grades in production at any given point in time are not perfect substitutes for the grade deliverable on the futures contract so that long hedgers will not be able to hedge all the risks that confront them. For an in-depth analytic description of this early literature, see Fort (1985).

[3] Houthakker gives two arguments as to why an excess of short hedging will dominate commodity futures markets. In the one not addressed here, he stresses the asymmetry of price arbitrage between long and short hedgers. While short hedgers face limited risk because the futures price cannot exceed the cash price by more than carrying charges, long hedgers have no such protection. This limited risk situation encourages short hedging relative to long hedging (Houthakker 1968, pp. 196–97). This conjecture concerning asymmetric arbitrage is analyzed in Lien and Quirk (1984), where it is shown that it holds only under rather restrictive conditions.

ing. During the later period of the crop year, when the basis is widening, there may be a contango (an expected *fall* in the futures price) or not depending on the relative strengths of the basis-widening effect and the low correlation effect.

We strive for an interpretation divorced from the anticipated basis movements that play such a large role in Houthakker's argument. The problem that we see with introducing anticipated basis changes into a theoretical account of backwardation is that this produces a "grin without the cat" theory. Short hedgers buy cash and sell futures in the immediate postharvest period because they believe that over the very near term less short hedging (relative to long hedging) will occur. But there is no explanation offered as to why this should be the case. Thus the anticipations aspect of the Houthakker theory explains the dominance of short over long hedging today as due to anticipations that short hedging will not be so dominant tomorrow, but (outside of further anticipations) why this should be so is outside the theory.

The link between net short hedging and this pattern of price correlation is that large inventories tend to be associated with low cash prices.[4] Since short hedgers endeavor to avoid the risks associated with low cash prices and the correlation between cash and futures prices is large with large inventories, the futures contract offers a desirable instrument for their purposes. On the other hand, long hedgers try to avoid the risks of high cash prices, but the low correlation between cash and futures prices during periods when cash prices are high limits the effectiveness of the futures contract for long hedging purposes. The result is that net short hedging characterizes the futures market.[5]

The objective of this paper is to spell out in some detail just how an inventory effect can be specified and its implications. Ultimately, the argument is based on the flexibility of futures contracts (as contrasted with forward contracts) and the lack of perfect substitutability among

[4] Cash prices, of course, are determined by quantities of a commodity demanded and supplied, not by inventories. But, given intertemporally stationary demand and assuming no carryover from one harvest to the next, a rational expectations equilibrium implies that quantities supplied to the market are directly related to inventory levels. Large releases to the market are required when inventories are large in order to ensure that price will rise sufficiently over the interharvest period to cover carrying costs.

[5] In a similar vein, Cootner (1960) argued that, when inventories are small, short hedging will be light, and if the output commitments of long hedgers are large, then hedging can be net long. The time when inventories are likely to be small is just before the harvest. Hence, Cootner expected a falling futures price until hedged inventories reach their peak (i.e., stocks in commercial hands are at their peak) and a rising price only after this peak. He concluded that the requirements for a rising futures price may not hold over a substantial portion of the duration of some contracts; there can be a period of net long hedging.

the various commodity grade-location options available for delivery under the futures contract. Our approach is as interesting as the question of backwardation itself since conditions under which a backwardation equilibrium will exist have received little rigorous treatment in the general way that we propose.

In Section II, we stress the important differences between forward and futures contracts. Our model of a true futures market is in Section III. In Section IV, we derive conditions under which an appropriately specified inventory effect can produce a backwardation equilibrium, even when short and long hedgers have the same probability beliefs and the same utility functions over profits. In this derivation, we find that Houthakker's original specification of an inventory effect falls short from the viewpoint of expected utility maximization. Conclusions round out the paper.

II. True Futures versus Forward Markets

A fundamental difference between commodity futures contracts and forward contracts is the flexibility provided to sellers (promising delivery) under the former.[6] For a wheat futures contract, the seller has the choice of the date during the delivery month to actually make delivery, the grade of wheat to actually deliver (at set penalties or premiums for nonstandard grades), and the delivery location itself (from a set of locations available under the contract). While this flexibility of delivery terms is essential for avoiding cornering problems and thinness of markets, it also creates uncertainty as to delivery terms for the buyer of a futures contract. For this reason, delivery rarely takes place under commodity futures contracts; cash contracts specifying delivery conditions in fine detail are typically used when the actual transfer of a commodity is contemplated.

This flexibility on the seller's side provides certain arbitrage relations that characterize the joint probability density function (pdf) between cash and futures prices at the delivery date. Because the seller of the contract has the choice of the grade-location combination to deliver, delivery of the lowest-priced alternative will occur should delivery become a reality. This ensures the following important arbitrage condition.

Assume that there are two cash grades, one delivery location under one futures contract, and both grades deliverable with no penalties or

[6] There are other differences between forward and futures contracts, including the lack of well-developed competitive markets for forward contracts, the fact that the profits and losses on futures contracts are paid out on a daily basis, and that a clearinghouse acts to protect futures traders from nonperformance, while no such protection exists in the case of forward contracts.

premiums in a two-period model denoted by subscript $t = 0, 1$. The prices of the two deliverable cash grades at time 1 are C_1^1 and C_1^2. The futures price at time t is denoted F_t. The following arbitrage relationship plays an important role in the derivation of the joint pdf of cash and futures prices:

$$F_1 = \min(C_1^1, C_1^2). \tag{1}$$

While set in a two-grade, single delivery location context, a more general condition is obvious. Relation (1) provides a basic rationale for the inventory effect and for Houthakker's explanation of backwardation. One way to state his idea of an inventory effect is that at low cash prices, the various grade-location options deliverable under a futures contract are closer substitutes for one another than at high cash prices because low cash prices occur when inventories of all delivery alternatives are large, and at such times, it is the common properties of the alternatives rather than their differences that determine their prices. Because the cash prices are more highly correlated with one another at low than at high cash prices, this implies from (1) that any cash price is more highly correlated with the futures price at low than at high cash prices.

It is instructive to consider the case of a forward market, in which perfect hedges are possible and in which the inventory effect is absent. If the futures market is a forward market defined by the absence of flexibility in delivery, then, in effect, the prices of the two cash grades are identically equal to each other at time 1. As a result, the cash and futures, or more properly spot and forward, markets "come together" at time 1: the forward market version of arbitrage condition (1). The difference between the time 1 spot and forward prices is zero, and there can be no effect on this zero price difference due to changing inventories. Adhering to a forward market analysis can tell us nothing about an important aspect of functioning futures markets since the joint pdf degenerates in this case.

III. Short and Long Hedging

Our model of a futures market in a two-period framework is this. Two types of hedgers are present in the futures market: elevator operators and millers. Elevator operators buy cash wheat today (in period 0) and store it for sale in period 1. To the extent that they hedge, elevator operators are short hedgers selling futures contracts to offset their long positions in the cash market.

The operation of long hedging by millers has been described in detail by Working (1953*b*). Millers make bids on flour contracts with flour users such as bakeries. For large flour contracts, wheat requirements are difficult to satisfy through immediate purchases in the oftentimes thin cash markets. As a consequence, the miller buys wheat futures at the time of a successful bid for a flour contract. As wheat is purchased over time to meet milling requirements, cash purchases are offset by corresponding sales of futures. Gradually, the initial long hedge is terminated. Millers thus are long hedgers who buy futures contracts to offset their short positions in the cash market.

We are interested in establishing the result that backwardation can emerge in a true futures market given an appropriately specified inventory effect, without resorting to assumed differences between long and short hedgers concerning attitudes toward risk, information, or size of cash commitments. Consequently, we will model the futures market as one in which there are, say, N elevator operators and N millers. All participants have identical utility functions and the same joint pdf's over cash and futures prices. We will also assume that they are all located at a common delivery point and deal in grade 1 wheat, with the grade-location combination an admissible delivery alternative under the futures contract.

Short hedgers (superscript S) are involved in productive transformation of the cash commodity.[7] A cash commodity revenue function $R(y^S)$ represents returns strictly related to activities concerning the commodity that are independent of changes in cash or futures prices, for example, commodity "grading" by elevator operators. The function $R(y^S)$ will be assumed strictly concave; that is, $R(0) = 0, R' > 0$, and $R'' < 0$. Elevator operators commit themselves to carry an amount of the cash commodity y^S between times 0 and 1, earning $C_1^1 - C_0^1 - k$ per unit carried, where k represents known costs of storage. They attempt to reduce the risk of changes in the value of their holdings by selling futures, earning $(F_0 - F_1)Q^S$ on their futures position, Q^S. Under the assumption that the commodity is perfectly nonperishable, the following expression defines the sum of production and futures trading revenues for short hedgers:

$$V^S = (C_1^1 - C_0^1 - k)y^S + R(y^S) + (F_0 - F_1)Q^S. \quad (2)$$

Long hedgers (superscript L) have a cash commodity revenue function, $R(y^L)$, that describes profits from the milling operation itself. It is assumed that the price quoted to the bakery on the flour bid (con-

[7] A more descriptive model of futures trading would include a multiplicity of contracts and an extended time period. Anderson and Danthine (1981) allowed for a multiplicity of contracts but under the special case of mean-variance analysis. Lien and Quirk (1984) applied a rational expectations framework to a T-period, single-contract model similar to the one developed here. They found that the futures market became a forward market in all periods prior to $T - 1$, indicating that one might just as well examine a two-period model.

verted to dollars per bushel of wheat) is the cash price of wheat at time 0 plus carrying costs, plus the markup from milling, $R(y^L)$. Payment for the cash commodity occurs at time 1. For this two-period model, all cash purchases of wheat by the miller have been telescoped into a single purchase at time 1, at which time the initial long hedge is terminated by an offsetting sale of futures contracts. For symmetry, we will assume that the $R(y^L)$ function is identical to the $R(y^S)$ function. Millers attempt to reduce the risk of changes in the cash price by buying futures, earning $(F_1 - F_0)Q^L$ on their futures position, Q^L. The following expression defines the sum of production and futures trading revenues for the long hedger:

$$V^L = (C_0^1 + k - C_1^1)y^L + R(y^L) + (F_1 - F_0)Q^L. \tag{3}$$

In order to incorporate these revenue functions into an expected utility framework, one must first derive the joint pdf between cash and futures prices. With the arbitrage relation in (1), the joint pdf over the futures price and the grade 1 cash price is

$$h(F_1, C_1^1) = \begin{cases} 0 & \text{for } F_1 > C_1^1 \\ \int_{C_1^1}^{\infty} f(C_1^1, C_1^2)\,dC_1^2 & \text{for } F_1 = C_1^1 \\ f(C_1^1, F_1) & \text{for } F_1 < C_1^1, \end{cases} \tag{4}$$

where $f(C_1^1, C_1^2)$ is the joint pdf over the time 1 cash prices. An exactly symmetrical story can be told for a joint density between the futures price and the grade 2 cash price. Since all participants are assumed to deal in grade 1 wheat, henceforth, let it be understood that C_1 and C_0 stand for the grade 1 cash prices at times 1 and 0, respectively, in order to ease the notational burden.

Houthakker argued that, because of the existence of multiple delivery alternatives under the futures contract, the joint density in (4) should be characterized by a high correlation between C_1 and F_1 at low values of the cash price and by a low correlation between these prices at high values of the cash price. The result is net short hedging and backwardation ($EF_1 > F_0$). In our treatment, an inventory effect must be interpreted in a somewhat different manner. On the basis of our expected utility maximization model, it appears that both cash *and* futures prices must be "low" in order for an inventory effect to generate backwardation. Further, in an expected utility framework, one finds that partial correlation coefficients aggregate price movements at a level that is too coarse for the purpose of deriving sufficient conditions for backwardation.

IV. Normal Backwardation and the Inventory Effect

In the case of a true futures market in which perfect hedges are not possible, there is a nondegenerate joint pdf over the cash and futures prices. To identify the role of the inventory effect in generating a backwardation equilibrium, it is convenient to postulate an initial martingale equilibrium with $h(F_1, C_1)$ symmetric about EC_1 and EF_1, equal cash commitments by long and short hedgers ($y^S = y^L$), and equal hedging by long and short hedgers ($Q^S = Q^L$). Then we examine the effect of perturbing the equilibrium.

In the case of a nondegenerate joint pdf, the objective function for the short hedger becomes

$$EU^S = \int_0^\infty \int_{F_1}^\infty u(V^S)h(F_1, C_1)dC_1dF_1 + \int_0^\infty u(V^S)h^*(C_1)dC_1, \quad (5)$$

where $h^*(C_1)$ is the pdf holding when C_1 is the minimum cash price, that is, $C_1 \equiv F_1$. The first term is expected utility occurring when C_1 is *not* the minimum cash price. For elevator operators, the first-order conditions with respect to y^S and Q^S (respectively) are

$$\begin{aligned} EU_y^S = &\int_0^\infty \int_{F_1}^\infty u'(V^S)(C_1 - C_0 - k + R')hdC_1dF_1 \\ &+ \int_0^\infty u'(V^S)(C_1 - C_0 - k + R')h^*dC_1 = 0, \end{aligned} \quad (6)$$

$$\begin{aligned} EU_Q^S = &\int_0^\infty \int_{F_1}^\infty u'(V^S)(F_0 - F_1)hdC_1dF_1 \\ &+ \int_0^\infty u'(V^S)(F_0 - C_1)h^*dC_1 = 0. \end{aligned} \quad (7)$$

These first-order conditions characterize the short hedger at an initial martingale equilibrium.

Integration of the short hedger's first-order conditions by parts yields

$$\begin{aligned} EU_y^S = &-\int_0^\infty \int_{F_1}^\infty H(F_1, C_1)[u''y^S(C_1 - C_0 - k + R') + u']dC_1dF_1 \\ &+ \int_0^\infty u'(V^S)(F_0 - C_1)h^*dC_1 = 0, \end{aligned} \quad (8)$$

$$EU_Q^S = -\int_0^\infty \int_{F_1}^\infty H(F_1, C_1)[u''y^S(F_0 - F_1)]dC_1 dF_1$$
$$+ \int_0^\infty u'(V^S)(F_0 - C_1)h^* dC_1 = 0, \tag{9}$$

where

$$H(F_1, C_1) = \int_{F_1}^{C_1} h(F_1, x)\, dx,$$

$$H^*(C_1) = \int_0^{C_1} h^*(x)\, dx.$$

With an appropriately chosen perturbation of the density h, a perturbation that incorporates an inventory effect as described presently, we will show that the volume of short hedging increases, while the volume of long hedging decreases, at the original martingale equilibrium. This means that the market-clearing price F_0 must fall (see fig. 1). Moreover, we can choose our inventory effect perturbation in such a way that EF_1 is unchanged. Since $F_0 = EF_1$ at the initial martingale equilibrium, the new equilibrium will exhibit backwardation, that is, $F_0 < EF_1$.

We proceed to this end as follows. First, we generalize the form of the joint pdf between cash and futures prices to incorporate a shift parameter. This allows us to impose an inventory effect directly onto the joint pdf. Second, we discuss the comparative statics of changes in cash commitment and hedging with respect to imposition of the inventory effect. Since we want net short hedging, finding the sign of the total derivatives of y and Q with respect to our inventory effect perturbation of the joint pdf h is the primary exercise. At this stage, we show that an inventory effect perturbation of the joint pdf helps to bring about increases in short hedging and decreases in long hedging. The result is that the futures price, F_0, must fall as short hedging increases relative to long. Then, all that remains is to show that EF_1 remains unchanged when we perturb the joint pdf. The result of the inventory effect perturbation will then be a backwardation equilibrium, $F_0 < EF_1$.

Notationally, perhaps the simplest way to express things is to view h as a function of a shift parameter α as well as of F_1 and C_1, that is,

$$h(F_1, C_1, \alpha) = h(F_1, C_1) + \alpha\theta(F_1, C_1), \tag{10}$$

where α is simply a shift parameter; for example, $h(F_1, C_1, 0) = h(F_1, C_1)$. We assume that ($a$) $\theta(F_1, C_1)$ satisfies

$$\int_0^\infty \int_{F_1}^\infty \theta(F_1, C_1)dC_1dF_1 + \int_0^\infty \theta(C_1, C_1)dC_1 = 0,$$

(*b*) $h(F_1, C_1) = 0$ implies $\theta(F_1, C_1) = 0$, and (*c*) α is restricted so that $h(F_1, C_1, \alpha) \geq 0$ for all (F_1, C_1).

Note that perturbing the joint pdf $h(F_1, C_1)$ is equivalent to imposing small changes on the joint pdf $h(F_1, C_1, \alpha)$ through small changes in α and evaluating the changes at $\alpha = 0$. Further, note that $\partial h/\partial\alpha$, evaluated at $\alpha = 0$, is $\theta(F_1, C_1)$.

Basically, we want to show how an inventory effect derived from a perturbation of the joint pdf encourages short hedging relative to long; that is, $dy^S/d\alpha$ and $dQ^s/d\alpha$ are both positive and α is consistent with our specific inventory effect. Performing comparative statics on the first-order conditions in (6) and (7) using the "shiftable" form of the joint pdf, we have

$$\frac{dQ^S}{d\alpha} = -\frac{(EU^S_{yy})(EU^S_{Q\alpha}) - (EU^S_{yQ})(EU^S_{y\alpha})}{\Delta}, \quad (11)$$

$$\frac{dy^S}{d\alpha} = -\frac{(EU^S_{QQ})(EU^S_{y\alpha}) - (EU^S_{yQ})(EU^S_{Q\alpha})}{\Delta}, \quad (12)$$

where $\Delta = (EU^S_{yy})(EU^S_{QQ}) - (EU^S_{yQ})^2$. Note that $\Delta > 0$ at a regular maximum. Further, second partials with respect to y and Q are negative for strictly concave utility. Since we are after conditions under which the total derivatives (11) and (12) will be positive, the signs of the cross-partial terms remain to be examined.

The inventory perturbation plays its role in the cross-partials with respect to α. We will need the inventory effect perturbation as discussed above and an appropriate specification of what is meant by "small" and "large" values of C_1 and F_1. First, the cross-partials with respect to α are

$$EU^S_{y\alpha} = -\int_0^\infty \int_{F_1}^\infty H_\alpha[u''y^S(C_1 - C_0 - k + R') + u']dC_1dF_1, \quad (13)$$

$$EU^S_{Q\alpha} = -\int_0^\infty \int_{F_1}^\infty H_\alpha[u''y^S(F_0 - F_1)]dC_1dF_1. \quad (14)$$

From (13), note that the term inside the brackets is negative (positive) when $C_1 \gtrless C_0 + k - R' + (1/\rho y)$, where $\rho = -(u''/u')$ is the coefficient of absolute risk aversion. Similarly, from (14), the term inside the brackets is negative (positive) when $F_1 \gtrless F_0$. These relationships give us the specification of large and small prices required for the inventory effect. It is an interesting discovery that there are requirements on *both* cash and futures prices. In terms of our model, Houthakker's original idea that cash prices high or low, alone, might drive backwardation falls short.

We are now ready to specify an inventory effect perturbation of the joint pdf so that both of the cross-partials are nonnegative and strictly positive if the perturbation involves strict inequalities over some intervals of positive probability measure. This effect will give the result that the probability that F_1 is "close" to C_1 is increased for small values of C_1 and F_1 while this probability decreases for large values of the two prices. Let $\partial h/\partial\alpha$ be denoted $h_\alpha(F_1, C_1)$. Note that (*a*) $h^*_\alpha(C_1) = \theta(C_1, C_1)$, (*b*) $H_\alpha(F_1, C_1) = \int_{F_1}^{C_1} \theta(F_1, x)dx$, and (*c*) $H^*_\alpha(C_1) = \int_0^{C_1} \theta(x, x)dx$. Consider the case in which (*a*) $H_\alpha(F_1, \infty) = H_\alpha(F_1, F_1) = 0$ for every F_1 and (*b*) $H^*_\alpha(C_1) = 0$ for all C_1. Using the specification of small and large found in (13) and (14), choose $H_\alpha(F_1, C_1) \geq 0$ when both F_1 and C_1 are small ($F_1 < F_0$ and $C_1 < C_0 + k - R' + [1/\rho y]$) and $H_\alpha(F_1, C_1) \leq 0$ when both F_1 and C_1 are large ($F_1 > F_0$ and $C_1 > C_0 + k - R' + [1/\rho y]$) with strict inequalities over intervals with positive probability measure for some terms. Note that this inventory effect follows the dictates of expected utility maximization since we require that *both* prices, C_1 and F_1, are small and large together in specifying the perturbation. With this specification of large and small prices and a perturbation of the joint pdf that induces an inventory effect, we also get the cross-partials with respect to α in (13) and (14) nonnegative and strictly positive over some intervals of positive probability measure. Thus we characterize an inventory effect as follows:

$$\Pr\{C_1 - F_1 \leq \epsilon | (C_1, F_1) \in R\} > \Pr\{C_1 - F_1 \leq \epsilon | (C_1, F_1) \in T\}, \quad (15)$$

where $R = [0, S)$ and $T = (S, \infty]$. The cutoff level for cash and futures prices is dictated by utility maximization: choose S so that $F_1 = F_0$ and $C_1 = C_0 + k - R' + (1/\rho y)$.

The effect of the perturbation can be seen from figure 2. By the basic arbitrage relation (1), $F_1 \leq C_1$. Given a small value of F_1, the dashed curve in figure 2*a* shows the cumulative density function (cdf) H plotted against C_1. The solid curve shows the portion of the perturbed cdf, which we call G in the figure, that differs from H following an inventory effect perturbation. The curve G is greater than H at low values of C_1, indicating that the probability that C_1 is close to F_1 is greater following the perturbation. Similarly in figure 2*b*, the perturbed cdf G is less than H at high values of C_1 so that the probability that C_1 is close to F_1 falls following the perturbation.

Our inventory effect perturbation signs some of the terms in (11) and (12), but we also need to worry about the remaining cross-partial derivative given by

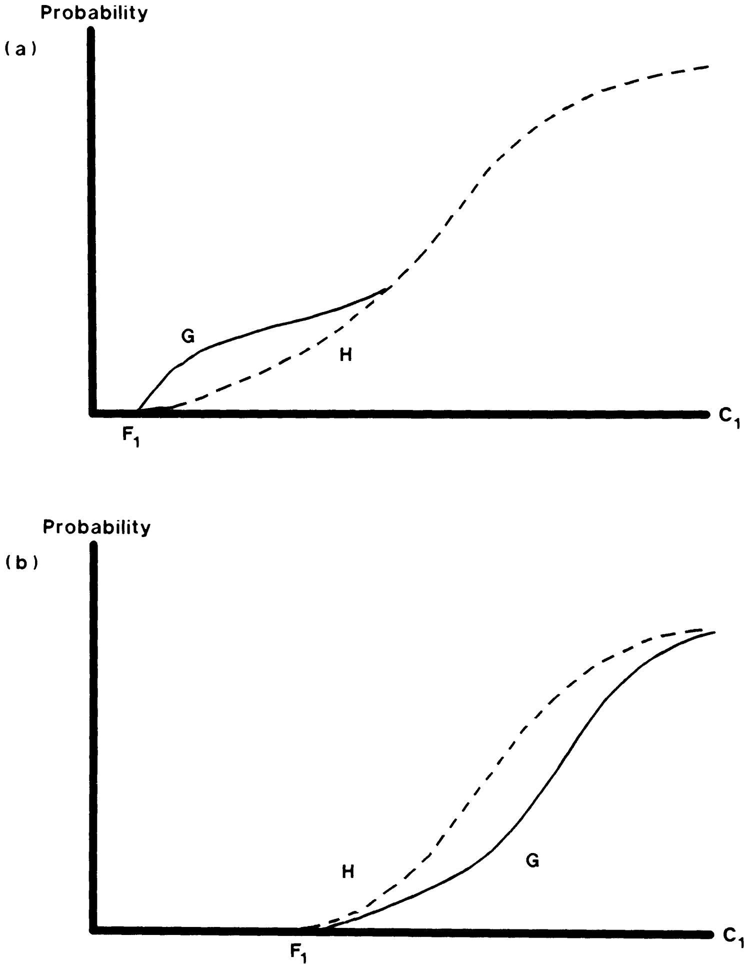

FIG. 2.—The inventory effect

$$EU^S_{yQ} = \int_0^\infty \int_{F_1}^\infty u''(V^S)(F_0 - F_1)(C_1 - C_0 - k + R')h(F_1, C_1)dC_1dF_1 + \int_0^\infty u''(V^S)(F_0 - F_1)(C_1 - C_0 - k + R')h^*(C_1)dC_1. \quad (16)$$

In general, (16) is of indeterminate sign. We simply postulate that Q and y are "complementary" in the sense that the marginal expected utility of hedging is an increasing function of the cash commitment of the trader so that the expression in (16) is positive.[8]

The end result, referring back to (11) and (12), is that $dQ^S/d\alpha$ and $dy^S/d\alpha$ are both greater than zero; short hedging and the size of the short hedger's cash commitment increase. This follows since all terms in (11) and (12) are positive except the second partials with respect to Q and y, which are negative. Applying the same perturbation to the long hedger's first-order conditions results in a *decrease* in both his hedged inventory and the size of his cash commitment. Thus the specified inventory effect perturbation of an initial martingale equilibrium with equal initial cash commitments by short and long hedgers induces an excess of short over long hedging at the old equilibrium price. The result is a decrease in the market-clearing futures price, F_0, given that speculative demand (supply) functions are of less than infinite elasticity.

All that remains is to show that EF_1 remained unchanged and a backwardation equilibrium will result from the inventory effect perturbation imposed on the joint pdf between cash and futures prices. That the perturbation has no effect on EF_1 can be seen by taking the following derivative:

$$\begin{aligned}\frac{\partial EF_1}{\partial\alpha} &= \int_0^\infty \int_{F_1}^\infty F_1 h_\alpha(F_1, C_1)dC_1 dF_1 + \int_0^\infty F_1 h_\alpha^*(F_1)dF_1 \\ &= \int_0^\infty F_1[H_\alpha(F_1, \infty) - H_\alpha(F_1, F_1)]dF_1 = 0 \end{aligned} \tag{17}$$

since $H_\alpha(F_1, \infty) = H_\alpha(F_1, F_1) = 0$ for every F_1 and $H_\alpha^*(C_1) = 0$ for all F_1 by hypothesis. Since the original joint pdf was symmetric, the per-

[8] In the case of a forward market, $EU_{yQ}^S > 0$ as long as F_0 is close enough to $C_0 + k - R'$ since if $F_0 - (C_0 + k - R') = \epsilon$, then EU_{yQ}^S can be written as

$$-\int u''(V^S)(F_0 - C_1)^2 h^*(C_1)dC_1 - \epsilon \int u''(V^S)(F_0 - C_1)h^*(C_1)dC_1$$

(integration from zero to infinity), which is positive for ϵ sufficiently small in absolute value. Things are more restrictive in the case of a true futures market. Let $\phi(F_1)$ be defined as

$$\int u''(V^S)(C_1 - C_0 - k + R')h(F_1, C_1)dC_1$$

(integration from F_1 to infinity) so that EU_{yQ}^S now becomes $\int (F_0 - F_1)\phi(F_1)dF_1$ (integration from zero to infinity) *plus* the expression above that holds for the forward case. If u is characterized by constant or decreasing absolute risk aversion and $EC_1 \leq C_0 + k - R'$, then $\phi > 0$. Signing EU_{yQ}^S requires further conditions on the derivative $\phi'(F_1)$. In particular, a sufficient condition for $EU_{yQ}^S > 0$ is $\phi'(F_1) \gtreqless 0$ for $F_1 \gtreqless 2F_0$, plus the condition given above for the forward case. One special case in which the cross-partial is positive occurs when h is uniform and symmetric about $EC_1 = C_0 + k - R'$ and u is a constant absolute risk aversion utility function, with $F_0 = EC_1$. But, in general, a positive cross-partial is a restrictive condition.

turbed pdf exhibits an inventory effect that leaves EF_1 unchanged, and with F_0 falling as short hedging increases, a backwardation equilibrium results. This is summarized in the following proposition.

PROPOSITION. In the case of a true futures market in which (*a*) all participants have identical strictly concave utility functions, (*b*) the futures market starts at an initial martingale equilibrium ($F_0 = EF_1$), (*c*) there are N long and N short hedgers, (*d*) all participants agree to the pdf over F_1, and (*e*) cash and hedged commodity commitments are complementary from an expected utility perspective ($EU^S_{yQ} > 0$), then for any common concave utility function there exists an inventory effect such that the resulting market equilibrium exhibits backwardation.

The work leading up to the summary proposition provides a set of sufficient conditions for an inventory effect to generate a backwardation equilibrium in a world with an equal number of identical short and long hedgers. The proof of the proposition utilizes only the qualitative properties of the first-order conditions so that less restrictive sufficient conditions can certainly be derived. However, they will be sensitive to the specific properties of the utility function assumed to characterize short and long hedgers. What the proof does make clear, however, is that an inventory effect depends not only on the level of the cash price but on the level of the futures price as well. This serves to emphasize that Houthakker's argument about the behavior of the basis relative to only the cash price is generally insufficient for backwardation, given the dictates of expected utility maximization. This is observed in the design of our inventory effect perturbation.

The proof also provides a further indication of why it is that we have interpreted the inventory effect in this paper as that the probability that cash and futures prices are closer together at low cash and futures prices is larger than when both prices are high. In contrast, defining the inventory effect specifically in terms of the properties of partial correlation coefficients at high versus low cash prices does not lend itself to simple proofs of backwardation. The reason for this is that correlation coefficients aggregate over ranges of cash and futures prices, and, as the proof indicates, a much finer kind of specification of the inventory effect is needed to prove backwardation for an arbitrary concave utility function.[9]

[9] The essence of our argument for backwardation, based on the inventory effect, is that the joint density over cash and futures prices is asymmetric in a special way. Because of the hypothesized asymmetry, this means that the mean-variance approach adopted in much of the literature dealing with futures markets (including a mean-variance approach in Houthakker's original paper) is inapplicable in analyzing the inventory effect since it is only in the case of an underlying normal (symmetric) distri-

In another paper, Fort (1986) provides a framework for investigating the empirical presence of the inventory effect based on expression (15). Note that this expression can be cast as a simple comparison of cdf's at low and high cash prices. Making such a comparison for March wheat contracts, 1968–82, Fort finds that such an effect is quite pervasive in observed futures price distributions. By our theoretical results and this related empirical analysis, the existence of backwardation again comes to the fore as an important element in the controversy over the ability of speculators to earn long-run profits.

V. Conclusions

In this paper, we have developed a set of sufficient conditions for the existence of backwardation in a true commodity futures market, on the basis of the notion of the inventory effect, as originally identified by Houthakker. The appeal of the inventory effect approach to backwardation is that it is based explicitly on the institutional features of operating futures markets, including especially the flexibility of futures contracts and the arbitrage relation linking cash prices to futures prices. Working with a true futures market model appears to be essential in developing a theory of price patterns on futures markets. Flexibility of delivery alternatives in futures contracts is a fact of life in these markets and introduces complications that can cause major modifications in theories developed for markets in forward contracts.

We have used a basic arbitrage relation to derive the joint density over cash and futures prices as of the maturity date of the futures contract and have examined the effects of introducing asymmetry into this density in the form of a special kind of inventory effect. What we have found is that it is useful to reformulate the notion of an inventory effect in a manner different from that of Houthakker's original formulation and that the mere presence of an inventory effect is not sufficient to establish a backwardation equilibrium: the inventory effect must be of a specialized type if backwardation is to be proved. We have explicitly avoided introducing into our model anticipated changes in the basis. As our proposition shows, it is possible to prove the existence of backwardation without introducing anticipations, which we feel is a weak link in Houthakker's original formulation.

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bution that a convincing argument can be made for the mean-variance approach. This gives one more indication that the current literature is in fact concerned primarily with a study of forward markets and not with true futures markets.

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