

Race, Technical Efficiency, and Retention: The Case of NBA Coaches

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Abstract

Despite the common perception that African-American coaches face discrimination obstacles, only two rigorous statistical studies exist that actually address the issue of racial variation in retention of coaches. Neither study accounts for variation in the level of talent across coaches and the production of wins. We examine the difference between retention of African-American and white NBA coaches based on technical efficiency calculations from stochastic production frontier estimates of team win production. First, we detect no difference in technical efficiency by race of the coach. Second, the evidence is consistent with the idea that coaches are retained based on their technical efficiency. Finally, the evidence fails to support any difference in retention of NBA coaches by race. These results offer finance insights for practitioners and raise methodology questions about retention discrimination findings in the NFL.

Keywords: Race, retention, stochastic production frontier analysis, NBA coaches.

Introduction

Our analysis pertains to employee retention by race, a small but important part of the overall analysis of racial bias in economic organizations. Others have examined player retention in the NBA (Hoang & Rascher, 1999) and in the NFL (Conlin & Emerson, 2006). We look, instead, at the coaching input and restrict our analysis to only race and retention, leaving the many issues surrounding initial hiring to future work (see Greenhaus, Parasuraman & Wormley, 1990, and Arrow, 1998, for discussions of discrimination in this broader context). It is practically taken

for granted that there is racial variation in the retention of professional team sport coaches. For example, in the face of general criticisms about the lack of minority head coaches, the NFL follows the "Rooney Rule." Owners have agreed in principle that any team seeking to hire a head coach would interview at least one minority candidate. The exception would be when a team makes a commitment to promote one of its assistants.

Interestingly, the same pretense was behind early academic analysis of the issue. Scully (1989) found a distinct bias in MLB toward hiring managers from the northern states who had been infielders in their playing days (pp.

179-181). He hastened to point out that African-American players were predominantly southern and typically not infielders. Singell (1991) found that sports-specific capital explains the decision to coach in the first place. Since star players were less sport-specific, there are few stars in the coaching profession. But on this dimension, since African-American stars had fewer general outlets, more of them would become coaches than white stars with similar career performance.

But these studies actually take discrimination as a given and suggest why it may occur. A very small literature directly addresses the issue of whether retention discrimination against African-American coaches actually can be detected. The economic logic guiding the analysis is straightforward. If there is no racial impediment in the “pipeline” that produces major league coaches, then, at the margin, there should be no difference in observed coaching ability by race. Consequently, there should be no detectable differences in the way that African-American coaches are hired and fired relative to their white counterparts.

The literature on this specific retention topic is small and with some shortcomings. Shropshire (1996) referred just to popular media accounts that show that the first three African-American Major League Baseball managers had better records with their teams than the white managers that followed them. Clearly, there is no “all else constant” element to these observations.

Madden (2004) examined the performance and retention of NFL coaches. She found that African-American coaches are hired by better teams, are more successful than their white counterparts during the regular season, and are only marginally less successful in the playoffs. Madden (2004) suggested that African-American coaches are being held to a higher standard to get and keep their jobs.

The uniformity of the results of comparing regular-season wins and participation in the playoffs for white and African-American coaches in various ways is striking. No matter how we look at regular-season success, African-American coaches have performed better. These data are consistent with African-Americans having to be better coaches than whites to be hired as a head coach in the NFL(p. 15).

In addition, she concluded that firing patterns suggest “last hire, first fire” rather than a careful consideration of success. But degrees of freedom issues plagued the study—there were only five African-American coaches in the NFL sample. So the author occasionally fell back on the lack of data as an explanation for “puzzling” outcomes (e.g., African-American NFL coaches were systematically likely to perform well in the regular season but then falter in the playoffs). But, actually, that same problem also casts statistical doubts on some of the outcomes touted by the author.

Finally, Kahn (2006) used a hazard function approach to estimate racial differences in retention probability, pay, and performance for NBA coaches. He found no support for the idea that black coaches faced discrimination in entry, pay, or retention. Although Kahn held team quality and payroll constant and employed indicators of coaching quality as explanatory variables, there was no formal modeling of the underlying production function for wins in order to identify *separately* the contributions of players and coaches.

And this is an important shortcoming. Unlike players, whose actual performances can be verified and evaluated by their playing statistics, the only data on coaching performance is winning record. But a coach’s winning record is, in turn, determined by the quality of the players that have been coached. In order to pass judgment on a coach’s effectiveness, the coach’s actual performance must be isolated from that of the players. This mission can be accomplished with technical efficiency (TE) comparisons derived from actual coaching outcomes and estimates using stochastic production frontier (SPF) analysis of the best that coaches have done with varying levels of team talent.

From the SPF perspective, a coach that can obtain the efficient level of winning, given the talent provided by the owner’s profit-maximization considerations on quality, would be “on the frontier” of winning production. TE measures the relationship between a coach’s actual winning achievement and the frontier. The closer a coach is to that level of winning, the more efficient. And here is an important strength of SPF analysis. A given coach can have *either a higher or lower TE* when offered the chance to coach the higher-quality team. The model includes the

real-world possibility that a given coach may manage lower-quality talent either more effectively, or less so, than higher-quality talent. With the same level of playing talent, if there is variation in coaching ability, winning records will vary across coaches and so will their TE measures.

The foregoing suggests the following. First, the NBA is a promising league for study because it has so many more African-American coaches than other leagues have. We do not mean to imply any expectation on our part about the eventual outcome, only that the NBA offers more observations to bolster the statistical integrity of our analysis of race and retention. Second, attention to the underlying production process, especially isolating the coach's role, covers more of the "all else constant" requirement than in past studies.

We add to the literature by applying the SPF methodology to the NBA to reduce the aforementioned shortcomings. Our approach is different from all others analyzing race impacts on retention in sports. First, in specifying the inputs for SPF analysis, the essential element of coaching is preserved—coaches manage *players*, not *player statistics*. Our measure of inputs concerns the contributions that player talents make to wins rather than, say, team shooting percentages across players. Second, we calculate coaching TE from SPF estimates of winning. These TE calculations account for the variation in team talent with which each coach has to work. We then take the novel step of analyzing the variation in coaching TE by race of the coach.

In our NBA example, holding TE constant, black NBA coaches are not fired or retained any differently than white coaches. Further, the lower TE of black coaches that do get fired explains what little variation there is in the tenure of black and white NBA coaches. That is, we do not find evidence of "last hire, first fire" in the NBA as Madden found for the NFL. This suggests further study of the variation in race and retention across pro sports league owners.

In addition, there are finance lessons for practitioners. First, TE is a useful measure that actually sorts coaches by success, holding the quality of their teams constant. Second, owners behave as if they use such a measure in their actual retention decisions; coaches are retained based on their winning efficiency. This casts doubts on

common criticisms of coaching choices coming from "good old boy" networks. Third, there are a very few coaches whose TE changes dramatically in our sample. Technically, SPF assumes this cannot be due to dramatic changes in player performance. But it could be due to dramatic roster alterations, and we find some evidence that this is true for those coaches. However, we cannot rule out by our approach explanations like the ever-elusive "team chemistry" explanation. Finally, coaches are paid millions, and mistakes can be costly. Our estimates show that keeping a poor coach could reduce winning efficiency by as much as 13%.

The paper proceeds as follows. In the next section, we detail our approach and specify the data. Section III contains the derivation of playing talent inputs for the SPF estimation, that is, our measure of marginal contributions to winning. Section IV details the empirical derivation of our TE results. In Section V, we use our TE estimates to analyze retention by race in the NBA. Lessons for practitioners and finance lessons are highlighted at the end of the section. In Section VI, the paper concludes that the method is insightful and there are challenges to be overcome in the literature claiming firing discrimination against black coaches. We stress again from the outset that our analysis pertains only to firing and retention, a small but important part of the overall analysis of racial bias.

Empirical Approach and the Data

Many studies have estimated SPF and team TE. Up-to-date work and an accompanying bibliography can be had from Dawson, Dobson, and Gerrard (2000a, 2000b), Haas (2004), Jewell and Molina (2004), Lee (2006), and Lee and Berri (2008). There also exists a literature specific to estimating the TE of coaches and managers. In pro sports, Kahn (1993), Singell (1993), and Scully (1994) followed the original contribution by Porter and Scully (1982). Horowitz (1994a, 1994b) used a simple statistical identity to assess a different form of managerial efficiency in MLB, and the limits of that approach were pointed out by Ruggerio, Hadley, Ruggerio, and Knowles (1997). In college sports, Clement and McCormick (1989) paved the way followed by Fizel and Ditri (1996, 1997).

Our approach is distinguished in three important ways from the past literature. First, neither of the papers cited earlier that focus on retention of professional coaches by race hold TE constant (Madden, 2004; Kahn, 2006). Second, none of the TE work just cited examined the impact of race. Third, Lee (2006) surveyed team efficiency estimation analyses as well as the SPF literature and found that only two other papers had coaches choosing playing talent rather than player statistics (Fizel & Ditri, 1996; Dawson, Dobson, & Gerrard, 2000a). Again, the present work calculates talent input contributions to winning on a player-by-player basis.

The most popular choice of input variables in the literature reviewed above has been playing statistics. For example, researchers have employed slugging percentage

and earned runs allowed in baseball, or shooting percentages, turnovers, and rebounds in basketball. We agree that those variables allow one to explain the variation in team win-loss records, and thus they are legitimate independent variables in a model designed to explain team wins or evaluate player performance.

But playing statistics are not legitimate input variables if one considers the standard microeconomic definition of production. In production, managers decide how best to combine inputs to maximize the profits of the firm. Every input has a positive marginal product, and the amount of inputs employed is exogenously determined by managers.

Playing statistics are not variables that a manager can choose exogenously to increase wins. In the sports industry, coaches adjust their line-up to increase the probabili-

Table 1. Player and Team Factors for the NBA. (Regular season aggregates across teams)

Variable	Notation	Mean	SD
Wins	WINS	39.378	13.848
Games Played	GM	78.755	9.676
Winning Percentage (WINS/GM)	PCT	0.500	0.163
Roster Stability	RS	0.678	0.134
Coach's Career Experience	CEXP	7.231	7.460
Coach's Career Winning Percentage	CPCT	0.508	0.112
Race	RACE	0.380	0.475
Player Factors			
Points Scored	PTS	96.868	4.992
Field Goals Attempted	FGA	80.777	3.426
Free Throws Attempted	FTA	25.575	2.445
Offensive Rebounds	RBO	12.641	1.493
Turnovers	TO	15.342	1.263
Defensive Rebounds	RBD	29.374	1.647
Steals	STL	8.149	0.998
Team Factors			
Team Rebounds	RBTM	5.292	0.563
Opponent's Points Scored	DPTS	96.868	5.228
Opponent's Field Goals Made	DFGM	36.461	2.436
Opponent's Free Throws Made	DFTM	18.977	1.991
Opponent's Turnovers	DTO	15.342	1.345

Source: Player and team factors are from Carter, 2003. Note: Tabled values for RS, CEXP, CPCT, and RACE are for the period 2000-01 through 2002-03. All other values are for the period 1993-94 through 2002-03, consistent with our use in the Appendix of estimates in Berri and Krautman (2006).

ty that their team will eventually prove victorious. Although these decisions impact the quantity of turnovers and rebounds a team may accumulate, it is not turnovers and rebounds the coach is choosing to expend or conserve. Additionally, the marginal products of playing statistics can often be negative. For example, turnovers in basketball have a negative impact on team wins.

In one last set of analyses, Hadley, Poitras, Ruggiero, and Knowles (2000), Haas (2003), and Kahane (2005) used payroll data as a talent measure rather than player statistics or the playing talent measurements we presently employ. In a perfectly competitive market, payroll is a legitimate proxy of playing talent. More importantly, payroll can capture intangible productivity or talent of a player that playing statistics cannot explain (for example, defensive skills, leadership, and big-game temperament). However, the fact that payroll is just playing talent multiplied by the price of talent requires care to separate the variation in price from the variation in payroll. One method is to use relative payroll scaled by the league average for a given season. But the limitation of this approach is that it requires the price of talent to be constant both over time and across teams, because payroll is just playing talent multiplied by the price of talent. We choose to measure player talent directly and leave analysis based on payrolls to others.

Our approach is to measure player talent directly by using various playing statistics. First, we obtain measures of the marginal contribution of a player's talent to winning. This step relies heavily on past work and existing data from other studies. These contributions, derived in the next section, are the inputs to the SPF analysis that follows.

In our second step, we detail the SPF model and empirical specification for the NBA case. Given the player talent provided to them, we determine the additional effectiveness of coaches in producing NBA wins. This is the approach that allows us to determine TE across the variety of coaches in our sample in Section V. In that last step, we also compare TE and retention across coaches by race.

Our chosen period for analysis is 2000-01 through 2002-03, and the data are as follows. For our first step, variables (and their descriptive statistics) used to derive player marginal contributions to winning are in Table 1 along with their descriptive statistics. Player marginal

contributions to winning are calculated for all players, 1999-2000 through 2001-02 because we use the previous season as the basis for expectations about the next season's performance (as explained in detail below). However, as detailed in Appendix A, part of our derivation of player marginal contributions to winning relies on player data over the period 1993-94 through 2002-03.

The remaining data required for our approach (the stability of team rosters, coaches' career experience lengths, coaches' winning percents at various points in time, and the race of coaches) were collected for the 2000-01 through 2002-03 seasons. In estimating the SPF model, there are 87 team observations, 29 teams each of the three seasons analyzed, 2000-01 through 2002-03. Of those observations, 38% involved a black coach. This does *not* mean that there were 33 (.38 x 87) different black coaches, as some black coaches coached multiple seasons. Indeed, the results of the SPF model generated TE measures for 47 head coaches, of whom 17 were black, 2000-01 through 2002-03. We note that only 38 different coaches coached complete seasons, and of these 14 were black.

Race of the coach was painstakingly verified by us from popular sources. Fort and Gill (2000) found that continuous measures of race and ethnicity, assessed by market makers rather than the researcher, generated statistically superior results for the case of MLB players. But the variation in race in the NBA is not so great (black and white, without dark-skinned Hispanics), and we suspect there is no bias in a dichotomous, researcher-assessed measure of race in our case.

Playing Talent Input Derivations

We determine the contribution that each of the players on a team make to winning, aggregated across positions, as the talent inputs that a coach has on hand to produce wins. Scully (1974) was the first to apply the idea of using statistics to explain wins and then determine player marginal contribution to winning. Others have done the same, including Berri (1999), who provided a literature review. But the explicit approach used here, determining player talents and the production of wins, was first introduced by Lee and Berri (2008).

Both player factors and team factors (labeled in Table 1) contribute to winning for any given team. To see the difference, consider field goals attempted (FGA) and opponent's field goals made (DFGM). The model we use (references are above and in Appendix A) is designed to connect team wins to the statistics tabulated for the individual players. The focus is on the individual players but to completely specify the model requires inclusion of factors that are tabulated only for the team. DFGM is needed to completely specify the model but, because it is not tabulated for individuals, it is treated as a team variable. FGA is tabulated for individual players, so it is credited to the individual player.

Generically, we write wins for team t as:

$$(1) \text{ Wins}_t = W(\text{PF}_t, \text{TF}_t)$$

where PF_t and TF_t are vectors of player factors and team factors for team t , respectively. Because players are the source of the PF_t variables, we are interested in finding how wins change with respect to player factors, that is, $\frac{\partial W}{\partial \text{PF}_t}$ for each of the variables in the PF_t vector. Each player i 's contribution to winning then becomes:

$$(2) C^i = \sum_{\text{PF}_t} \left(\text{PF}_t^i \times \frac{\partial W}{\partial \text{PF}_t} \right),$$

that is, the sum of all PF_t contributions made by a particular player. Following Scott, Long, and Sompil (1985) and Berri (1999), we distribute the team factors, TF_t , to each player proportional to playing time. If player i 's share of team total minutes played is m^i , then his allocation of TF_t is:

$$(3) S^i = m^i \sum_{\text{TF}_t} \frac{\partial W}{\partial \text{TF}_t}$$

With the groundwork in (1) through (3), each player's marginal contribution to winning is:

$$(4) \text{ MW}^i = C^i + S^i$$

A player's per-minute performance is augmented (or reduced) by the per-minute team factors.

For season t , we use MW^i on a per-minute basis from season $t-1$; the best guess for this year's performance is how players performed last year. If the player did not play in the immediate prior season, then his productivity was taken from his most recent season. Rookies pose a special challenge because they have no talent demonstration at the NBA level at all. We constructed the average level of productivity a team could expect given the player's draft

position. Such averages were calculated for players drafted in positions 1-10, 11-20, 21-29, 30-39, and 40-57. Additionally, averages were calculated for rookie players who were not drafted. Given our measure of talent per minute, we take the minutes from the current season to estimate how many wins each player was expected to offer in the current campaign.

As mentioned earlier, we choose to aggregate these contributions by position. This is somewhat arbitrary, but it: (1) combines players who are similar in performance, (2) conserves on degrees of freedom, and (3) moves us to the level of analysis across teams, as opposed to across players (each team now has three variables for each season). So the guard (GD), small forward (SF), and "big man" (BM) talent that a coach has to work with are given by:

$$(5) \text{ GD} = \sum_{i \in \text{GD}} \text{MW}^i$$

$$(6) \text{ SF} = \sum_{i \in \text{SF}} \text{MW}^i$$

$$(7) \text{ BM} = \sum_{i \in \text{BM}} \text{MW}^i$$

Table 2 shows the calculations of $\frac{\partial W}{\partial \text{PF}_t} \forall \text{PF}_t$ and $\frac{\partial W}{\partial \text{TF}_t} \forall \text{TF}_t$ (details of the calculations are in Appendix A). *Seriatim*, the estimates in Table 2 are used to calculate the results specified in (2) through (4). Finally, the player talents used in our SPF analysis are calculated according to (5) through (7). Note that our GD, SF, and BM input variables are of *ex ante* playing talent inputs by position before a season begins. Therefore, our measure of technical efficiency includes not only a direct impact of a coach on team-winning production by coaching a game, but also an indirect coaching impact through training and motivating (Dawson, Dobson, & Gerrard, 2002; Lee & Berri, 2008).

SPF Analysis and the Empirical Derivation of Technical Efficiency

The technical relationship between SPF and technical efficiency is in Appendix B, where it is made clear how to specify the production of team wins. For team t in our NBA case, we choose the following production function specification:

$$(8) \ln\left(\frac{\text{Wins}_t}{\text{SF}_t}\right) = \beta_0 + \beta_1 \ln\left(\frac{\text{GD}_t}{\text{SF}_t}\right) + \beta_2 \ln\left(\frac{\text{BM}_t}{\text{SF}_t}\right) - u_t + v_t$$

Table 2. Marginal Winning Product from Player and Team Factors.

Player Factors	Marginal Value
PTS	0.033
FGA	(0.034)
FTA	(0.015)
RBO	0.034
TO	(0.034)
RBD	0.034
STL	0.034
Team Factors	Marginal Value
DPTS	(0.033)
DFGM	0.034
DFTM	0.015
DTO-STL	0.034
RBTM	0.034

Source: See Appendix A.

Table 3. Descriptive Statistics

	Min	Max	Median	Average	Std.Dev.
Win	15.00	61.00	43.00	41.00	11.88
GD	0.18	0.68	0.42	0.41	0.11
SF	0.08	0.36	0.19	0.19	0.06
BM	0.24	0.58	0.38	0.38	0.08
RS	0.27	0.90	0.69	0.68	0.13
CEXP	0.00	29.00	4.24	7.23	7.46
CPCT	0.26	0.75	0.52	0.51	0.11

comes from the version of expression (1), above, detailed in Appendix A. SF_t , GD_t , and BM_t are the summed talent inputs across small forwards, guards, and big men, respectively, described in the last section. And we have a panel of teams across three seasons, 2000-01 through 2002-03. The results of statistical testing (not reported here but available upon request) fail to reject either the Cobb-Douglas production function, compared to the more flexible translog alternative, or constant returns to scale. So we estimate (8) as Cobb-Douglas with constant returns to scale by maximum likelihood.

Appendix B also shows the theoretical derivation of the TE equation specified for our NBA coaches as:

$$(9) u_t = \delta_0 + \delta_1 RS_t + \delta_2 CEXP_t + \delta_3 CPCT_t + \delta_4 RACE_t + w_t,$$

and, again, this is a panel over three seasons. Exogenous variables that may influence TE are roster stability (RS), the coach's career experience (CEXP), and the coach's career winning percentage (CPCT). The measurement of CEXP and CPCT are straightforward. RS is a form of weighted average of playing time for players appearing in the current and last season (the precise calculation is in Berri, Schmidt, & Brook, 2004). Finally, we also come to the variable that is the focus of our analysis, namely, the coach's race (RACE = 1 if African-American, 0 else). Table 3 includes the descriptive statistics for all of the variables in (8) and (9).

Table 4 shows the results of the single-step maximum likelihood estimation process for (8) and (9). Models 1 through 5 in Table 4 are offered in order to ascertain the importance of the chosen exogenous variables on TE. All of these exogenous variables are included in Model 1 and then singly in Models 2 through 5. We find that RS and CPCT are statistically significant, but not CEXP. And there isn't any support for variation in TE by RACE. Given this, we use the results in Model 6 that include only statistically significant variables. As an additional veracity check, we compared actual wins to the level of wins predicted from our estimates in Model 6. The correlation between predicted win production in (8) and actual team wins is 0.983 and, of course, the predicted average number of wins was 41 for an 82- game NBA season.

The BM and GD contributions are approximately the same and more than twice that of the SF position. Further, the coefficient estimates for both RS and CPCT coefficients are negative. So, greater roster stability or a higher career winning percent for the coach generates greater TE. The size of the roster stability impact is just the coefficient δ_1 . For a team with TE = 0.70 and a coach with a career winning percentage of 0.500, hiring a coach with career winning percent of 0.600 increases TE to 0.79.

The TE results in Table 4 generate two more interesting hypothesis tests. First, the routine test for an SPF model rejects that the TE effects are not stochastic because the data reject that $\gamma=0$ in our chosen Model 6. So the model does not reduce to a traditional average production

Table 4. Production Function Estimation Results

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Constant	5.048 (78.66)	5.011 (84.27)	5.073 (88.66)	5.013 (82.14)	5.027 (81.89)	5.046 (79.57)
ln GD	0.408 (8.30)	0.412 (8.23)	0.411 (8.36)	0.408 (8.10)	0.413 (8.60)	0.410 (8.64)
ln SF	0.171	0.119	0.167	0.125	0.146	0.170
ln BM	0.421 (5.82)	0.469 (6.70)	0.422 (6.29)	0.467 (6.45)	0.441 (6.34)	0.420 (5.91)
δ_0	1.766 (4.13)	-0.792 (-0.58)	1.314 (1.80)	-0.349 (-0.40)	1.378 (3.19)	1.800 (4.86)
RS	-1.612 (-2.53)		-1.852 (-2.70)			-1.595 (-2.65)
CEXP	0.001 (0.07)			-0.038 (-1.00)		
CPCT	-1.280 (-1.62)				-3.065 (-2.08)	-1.340 (-2.09)
RACE	0.042 (0.31)	0.258 (0.67)				
δ^2	0.112 (2.55)	0.381 (1.03)	0.134 (1.17)	0.346 (1.13)	0.185 (1.88)	0.113 (2.54)
γ	0.923 (19.62)	0.979 (47.42)	0.952 (52.30)	0.976 (43.41)	0.955 (30.82)	0.923 (21.16)
ln L	11.800	-1.525	8.253	-0.449	5.696	11.750
$H_0: \gamma = 0$	46.38 [0.00]	19.72 [0.00]	39.28 [0.00]	21.88 [0.00]	34.17 [0.00]	46.28 [0.00]
$H_0: \delta_1 = \dots = \delta_g = 0$	27.23 [0.00]	0.58 [0.45]	20.13 [0.00]	2.73 [0.10]	15.62 [0.00]	27.13 [0.00]

Notes: As stated in the text, there were 87 team observations. t-values are in parentheses; p-values are in square brackets. Given the specification of equation (8), the estimated coefficient of ln SF is .

model, and our use of the *stochastic* frontier is justified. Note further that all estimates of γ are close to 1, which indicates that the TE effects are likely to be highly significant in the analysis of NBA team output.

Next, by our Model 6, we reject that $\delta_1 = \dots = \delta_g = 0$, that is, we reject that the TE effects are not a linear function of RS and CPCT. Actually, this hypothesis is rejected in the models in Table 3 only when RS and/or CPCT are included. When they are not included in the TE equation, we

fail to reject this hypothesis. This further reinforces our choice of Model 6 in the first place.

TE, Retention, and Race

We take our first look at race and TE for coaches that were fired, 2000-01 through 2002-03, plus the coaches for 2003-04, in Table 5. Although the line between “quit” and “fired” can be a thin one, coaches who quit, officially, were not included in Table 5. (Rudy Tomjanovich left the Rockets due to health reasons; Pat Riley quit the Heat;

Table 5. Technical Efficiency of Fired Coaches

Team	2000-01		2001-02		2002-03		2003-04
	Coach	TE	Coach	TE	Coach	TE	Coach
Atlanta	Kruger	0.585	Kruger	0.679	Kruger-Stotts	0.718	Stotts
Charlotte	Silas*	0.884	Silas*	0.777	Silas*	0.892	Floyd
Chicago	Floyd	0.332	Floyd- Cartwright*	0.498	Cartwright*	0.622	Cartwright*
Cleveland	Wittman	0.512	Lucas*	0.555	Lucas* - Smart*	0.519	Silas
Denver	Issel	0.833	Issel-Evans*	0.609	Bzdelik	0.385	Bzdelik
Detroit	Irvine	0.610	Carlisle	0.963	Carlisle	0.875	L. Brown
G. State	Cowens	0.403	Cowens- Winters	0.431	Musselman	0.934	Musselman
Indiana	Thomas*	0.665	Thomas*	0.837	Thomas*	0.897	Carlisle
LA C.	Gentry*	0.627	Gentry*	0.821	Gentry- Johnson*	0.598	Dunleavy
Milwaukee	Karl	0.950	Karl	0.795	Karl	0.857	Porter
Phoenix	Skiles	0.824	Skiles- Johnson*	0.585	Johnson*	0.873	Johnson*
Portland	Dunleavy	0.739	Cheeks*	0.795	Cheeks*	0.847	Cheeks*
Toronto	Wilkins*	0.862	Wilkins*	0.736	Wilkins*	0.492	O'Neill
Vancouver	Lowe*	0.530	Lowe*	0.508	Lowe* H. Brown	0.558	H. Brown
Washington	Hamilton*	0.820	Collins	0.468	Collins	0.897	Jordan
		Average TE					
Entire League		0.760					
At Firing: All Coaches		0.670					
At Firing: Black Coaches		0.676					
At Firing: White Coaches		0.666					

Notes: TE = technical efficiency. *Denotes a black coach. For 2003-04, coaches listed for the beginning of the season.

Jeff Van Gundy quit the Knicks; Larry Brown quit the 76ers.) Because TE is calculated on a season basis, in seasons in which a coaching change occurred, the TE listed is for both coaches, combined.

Table 5 also includes overall league-average TE (fired and retained coaches), average TE of all fired coaches, and average TE at firing broken out by race. In calculating averages, we took the following approach for coaching changes: If a coach was fired at the end of a season, his TE

in that season was used for calculation of average TE of the fired coaches. If a coach was fired during a season, and later than the halfway point, then we just counted TE in that season in the average. But if he was fired before a half of a season passed, then a weighted average TE was used. For example, Atlanta Coach Kruger was fired one-third of the way through the 2002-03 season. Let $t = 2002-03$ and $t - 1 = 2001-02$. His weighted average is calculated as follows:
$$\frac{TE_{t-1} + (0.33 \times TE_t)}{1 + 0.33} = \frac{0.679 + (0.33 \times 0.718)}{1.33} = 0.689.$$

Table 6. Technical Efficiency of Retained Coaches

Team	Coach	Tenure	TE		
			2000-01	2001-02	2002-03
Charlotte	Silas*	4.5	0.884	0.777	0.892
Dallas	Nelson	5.5	0.964	0.898	0.934
Houston	Tomjanovich	11.5	0.941	0.528	0.920
Indiana	Thomas*	3.0	0.665	0.837	0.897
LA Lakers	Jackson	4.0	0.897	0.946	0.773
Miami	Riley	7.0	0.852	0.714	0.523
Milwaukee	Karl	5.0	0.950	0.795	0.857
Minnesota	Saunders	7.5	0.866	0.908	0.928
New Jersey	Scott*	3.0	0.528	0.850	0.899
Orlando	Rivers*	4.0	0.863	0.747	0.805
Philadelphia	L. Brown	6.0	0.892	0.802	0.837
Sacramento	Adelman	5.0	0.872	0.912	0.894
San Antonio	Popovich	6.0	0.886	0.905	0.952
Toronto	Wilkens*	3.0	0.862	0.736	0.492
Utah	Sloan	17.0	0.912	0.745	0.798
		Average Tenure	Average TE		
Black Coaches		3.50	0.782		
White Coaches		7.45	0.853		
All Coaches		6.11	0.830		

Notes: TE = technical efficiency. *Denotes a black coach. Fired after 2002-03: Silas, Thomas, Karl, and Wilkens.

And this amount was used in the calculation of the appropriate averages at the bottom of Table 5.

The results in Table 5 suggest the following. First, firings appear to occur as if owners use TE in the decision. The league average TE is about 13% higher than the average TE of fired coaches (0.760 > 0.670), so that below-average TE coaches were fired. Further, white coaches and African-American coaches (marked with an asterisk in Table 5) that were fired can't be distinguished on the basis of TE (0.676 for black coaches compared to 0.666 for white coaches, about 1.5%). This actually isn't surprising given that RACE was insignificant in our earlier TE estimation.

Table 6 shows TE for coaches who were retained over the same period. Average tenure and TE also are shown for all coaches and are broken out by race. First, we note

that the average TE of white coaches who were retained is 12% higher than the league average across all coaches in Table 5 (0.853 > 0.760), and retained African-American coaches also were above the league average but only marginally (0.782 > 0.760, just under 3%). Further, the TE of retained black coaches is also above that of fired black coaches (0.782 > 0.676, or nearly 16%). As with the previous evidence on fired coaches, this evidence on coaches that held their jobs is consistent with the idea that retention is based on TE. And, notably, the lower tenure period for African-American coaches is consistent with being above the league average TE, but lower than the TE exhibited by white coaches.

After the 2002-03 season, white coach Karl (TE = 0.867 and 5-year tenure) was fired, as were African-American coaches Silas (TE = 0.851 and 4.5-year tenure), Thomas

(TE = 0.800 and 3-year tenure), and Wilkens (TE = 0.697 and 3-year tenure). The firing of Wilkens, with below league average TE and truly disastrous TE for 2002-03, follows our previous observation that TE matters in the firing decision. Further, although Silas and Thomas did have TE above the league average (by 12% and 5%, respectively), they did have lower TE than the average of white coaches who kept their jobs up to 2002-03. Again, by and large, these findings do not show evidence of racial bias in firing. Milwaukee's firing of Karl remains an anomaly because his average TE is above even his white cohorts.

In closing, we offer the following finance lessons for practitioners. First, TE is a useful measure because it: (1) actually sorts coaches by their contribution to team success and (2) is also flexible enough to allow comparisons across various groups for any practitioner reason. Second, owners behave as if they use such a measure in their actual retention decisions. This will come as news to some observers adhering to other models of retention (e.g., good old boy networks). Third, NBA coaches are highly paid, so knowing when to replace a coach is important financially. In fact, fired coaches had a TE that is 13% lower than retained coaches with clear significant implications for paying coaches. Owners know the value of winning sold to fans, and they can calculate the marginal contribution of coaches through TE assessments.

As a caveat for practitioners, there is more to retention than meets our model's eye—we refer to the case of Tomjanovich in Table 6. Our SPF model uses *ex ante* player productivity as inputs (productivity measure in a previous season or average productivity measure in previous three seasons). But coaches have both direct effects and indirect effects (through training and motivating players, or for different team chemistry) so we are assuming that what a player did in the past he will do in the future *if coaching indirect effects are constant over seasons*. The volatility of TE under Tomjanovich (we found no instances of important player injuries) may reveal that these indirect effects may be quite important *for a few coaches*. But we did also find that roster stability is a statistically significant contributor to TE. Under Tomjanovich, RS also went from 0.876 to 0.662 to 0.747. The drop in RS in his second season gives a 34% drop in TE from our estimated coefficients. Thus, whether it is indirect effects, not

included in our modeling technique, or just roster instability remains a topic for future analysis.

Conclusions

The sources and impacts of discriminatory practices are many and varied. In this paper, our specific focus is on discrimination in retention of NBA coaches. Only two papers actually attempt to detect retention variation by race, and what little work there is has some shortcomings that this paper tries to remedy. We analyze the relatively data-rich NBA and use stochastic production frontier analysis to assess firing and retention by race, holding the technical efficiency of NBA coaches constant. In addition, in our approach coaches manage player talent, measured by player marginal contributions to winning, rather than playing statistics. There are three main findings: (1) there is no detectable difference in technical efficiency by race of the coach, (2) the evidence is consistent with the idea that coaches are retained based on their technical efficiency, and (3) the evidence fails to support any difference in firing and retention of NBA coaches by race.

This is in stark contrast to Madden's (2004) findings on the NFL but consistent with Kahn's (2006) findings on the NBA. Since Madden suggests that owner bias is the most likely source, the insight for the NFL may be as simple as this. Fan racial preferences, even if different than NFL owner preferences, are not strong enough to put a high enough economic cost on owner discriminatory retention practices.

But the answer is different in the NBA. On the one hand, fan and owner preferences may coincide so that technical efficiency really is all that matters among NBA coaches. On the other hand, if NBA owners are prejudiced, fan preferences put a high enough cost on those preferences that owners cannot exercise them. But investigating which is true, and just why there is a difference between the NFL and NBA situations, remains for future study.

We also find finance lessons for practitioners. TE sorts coaches by their contribution to team success, owners actually appear to behave as if they use such a measure in their retention decisions, and TE is an important element in calculating just how much to pay a coach. But practitioners should also be aware that our findings do

not necessarily rule out other types of contributions, such as training and motivating players or managing team chemistry.

As a final caveat, we offer this. Any result that fails to find evidence of discrimination in retention is only a small part of the overall analysis of discriminatory impacts. This does not explain why or under what circumstances discrimination might take place. In fact, given the existing research (Cunningham, Bruening, & Straub, 2006; Kahn, 2006), one might expect, a priori, that race would not influence personnel decisions within the NBA. At least we are able to offer the insight that coaching retention in the NBA is not the place to look for the exercise of racial discrimination.

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Acknowledgement

The second author acknowledges that this research is financially supported by the 2007 Sogang University Research Fund.

Appendix A: Player Factors and Team Factors Marginal Impacts

Our derivation of the marginal effects in Table 2 is as follows. First, Berri and Krautmann (2006) designed the following version of (1):

$$\text{Wins}_t = \alpha_0 + \alpha_1 \left(\frac{\text{PTS}_t}{\text{PE}_t} \right) + \alpha_2 \left(\frac{\text{DPTS}_t}{\text{PA}_t} \right) + \varepsilon_t,$$

where PTS are points, DPTS are opponent points, and:

$$\text{PE} = \text{FGA} - \text{RBO} + \text{TO} + 0.4\text{FTA},$$

$$\text{PA} = \text{DTO} + \text{RBD} + \text{RBTM} + \text{DFGM} + 0.4\text{DFTM}.$$

The PE and PA functional forms are the result of a long and extensive literature evolution, begun by Hollinger (2003) and Oliver (2003), detailed and extended in Berri and Krautmann (2006), followed by Berri, Schmidt, and Brook (2006), Berri and Schmidt (2007), and Lee and Berri (2008). The OLS results of estimating (1) from Berri and Krautmann (2006, p. 543) were:

$$\text{Wins}_t = 0.495 + 3.122 \left(\frac{\text{PTS}}{\text{PE}} \right) - 3.118 \left(\frac{\text{DPTS}}{\text{PA}} \right).$$

The effects in Table 2 involve the derivatives of with respect to all of the variables in PE and PA. To calculate these derivatives one uses both the estimated coefficients from Berri and Krautmann, above, and the mean values of PTS, PE, DPTS, and DPA. Each of the factors that comprise PTS, PE, DPTS, and DPA are listed in Table 2. The lone exception is steals. If the opponent commits a turnover and one can identify the player on the team who is responsible, the identified player is given credit for a steal. So the opponent's turnovers include all of the steals each individual player on the team garners. In Table 2 the value of a steals are listed among statistics tabulated for players, and the opponent's turnovers that are not steals are listed among team variables. One should also note that a small number of turnovers is not recorded for individual players. One could therefore add team turnovers to the list of team factors in Table 2.

Here are two examples to give a flavor for all of the results in Table 2. First, from the Berri and Krautmann formulation above, $\frac{\partial \text{WINS}}{\partial \text{PTS}} = \alpha_1 \left(\frac{1}{\text{PE}} \right)$. If the estimate of $\alpha_1 = 3.122$ and the mean of PE using the average values in Table 1 and the formulation of PE above is 94.6, then $\frac{\partial \text{WINS}}{\partial \text{PTS}} = 0.033$, as in Table 2. Similarly, $\frac{\partial \text{WINS}}{\partial \text{FTA}} = \frac{\partial \text{WINS}}{\partial \text{PE}} \frac{\partial \text{PE}}{\partial \text{FTA}} = 0.445 \times \alpha_1 \left(\frac{1}{\text{PE}} \right) \left(\frac{\text{PTS}}{\text{PE}} \right) = -0.034$, also at the means of PE and PTS. And the rest of the entries in Table 2 were derived in the same fashion.

Appendix B: SPF Derivation of Technical Efficiency

The SPF model for panel data is defined by:

$$(8') y_{it} = \alpha + x_{it}\beta + v_{it} - u_{it}$$

where y_{it} is the dependent variable that represents the logarithm of output at the period t ($t = 1, \dots, T$) for firm i ($i = 1, \dots, N$), x_{it} is k vector of inputs, β is a $k \times 1$ vector of coefficients, and v_{it} is an i.i.d. $N(0, \sigma_v^2)$, independently distributed of the u_{it} . The logarithmic form follows all of the development in the literature on the SPF. The variable u_{it} is the non-negative “technical inefficiency” error. Expression (8') is the theoretical foundation of our empirical specification in (8) in the text.

Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977) introduced, independently, the SPF model, following Farrell's (1957) definition of relative production efficiency. The central goal of this approach was to resolve the conflict between available data sets and the definition of a production function. The output data we observe are smaller than, or equal to, the maximum possible quantities that can be produced due to the existence of technical inefficiency. But a production function, by theoretical definition, specifies the maximum possible quantity of output given the quantities of a set of inputs. SPF models resolve this conflict by constructing a regression production function with two error terms as defined above, one representing the production loss caused by technical inefficiency, which is smaller than or equal to zero, and the other representing statistical noise.

Here, the inefficiency error is assumed to be independently distributed such that u_{it} is obtained by truncation at zero of the normal distribution with mean $z_{it}\delta$ and variance σ_u^2 . The $1 \times g$ vector z_{it} is a set of exogenous variables that affect technical inefficiency and δ is a $g \times 1$ vector of coefficients.

Given this, the technical inefficiency effects u_{it} are specified as

$$(9') u_{it} = z_{it}\delta + w_{it}$$

The random variable w_{it} is defined by the truncation of the normal distribution with zero mean and variance σ_u^2 . In the Battese and Coelli (1995) model $w_{it} \geq -z_{it}\delta$, consistent with the requirement that u_{it} is a non-negative truncation of $N(z_{it}\delta, \sigma_u^2)$. This is the so-called technical efficiency “effects” model since the additional equation (9') includes exogenous factors affecting TE. Expression (9') is the theoretical foundation for expression (9) in the text.

There are two ways to estimate the production function (8') and the effects function (9'). One is to estimate the production function first and then estimate the effects function, a two-step approach. Another approach is to estimate both equations simultaneously, a one-step approach. It has been recognized since the early 1990s that the one-step approach overcomes specification error plaguing two-step approaches in analysis of exogenous factors affecting TE. Kumbhakar, Ghosh, and McGuckin (1991), Reifschneider and Stevenson (1991), and Battese and Coelli (1995) developed the one-step method that solves the specification error and its resulting biased estimates. Wang and Schmidt (2002) provide a useful review of this topic, including extensions showing that estimation bias of production function coefficients and technical efficiency are large by simulation methods.

The difference in the aforementioned one-step model is the assumption in (9') that makes the technical inefficiency term, u_{it} , positive. Battese and Coelli (1995) developed a one-step model within a panel data context that has been applied to various empirical analyses. We apply their model to our NBA data and allow the outcome to vary by race in order to address directly the issue of variation in retention by race, after TE is controlled.

The likelihood function is derived by Battese and Coelli (1995), and it is expressed in terms of the variance parameters $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\gamma = \frac{\sigma_u^2}{\sigma^2}$. Battese and Coelli (1995) derived the likelihood function of $\varepsilon_{it} = v_{it} - u_{it}$ and the con-

ditional expectation formula of technical inefficiency, $E[u_{it}|\varepsilon_{it}]$. Additionally, technical efficiency is the ratio of actual output and the frontier output, that is:

$$TE_{it} = \frac{\exp(y_{it})}{\exp(\alpha + x_{it}\beta + v_{it})} = \exp(-u_{it}) = \exp(-z_{it}\delta - w_{it}).$$

Note that we take the exponential because y_{it} is the log-form of output, suggesting that maximum likelihood estimation be applied.