Optimal Competitive Balance in a Season Ticket League

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Abstract:

Theory predicts that a planner maximizing the sum of fan and owner surpluses from a league dominated by season ticket sales may prefer more, less, or the same level of competitive balance produced by a league of profit maximizing owners. Which it is depends on the relationship between marginal impacts of talent rearrangements in larger-revenue and smaller-revenue markets. Ultimately, then, judging whether an increase in balance enhances welfare rests on careful and thorough empirical investigation. Our reading of the literature and the policy debate shows that this careful work remains to be done.

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There is no question the level of play has decreased. Now, do games become more exciting? Are teams more evenly matched? No question. Is that good for the game or not? I don't know. I really don't know. I ask that question all the time.
—NFL Hall of Fame Quarterback Troy Aikman, quoted in Pedulla (2003).

I think that margin of competition, that margin of the difference between winning and losing in this league is very small, and I think that is great for the fans because every team comes in with an opportunity to win.
—NFL Commissioner Roger Goodell, quoted in Curran (2008).

I. Introduction

In this paper, we lay out the basic welfare foundation of optimal competitive balance for regular season play in a “closed” sports league where season tickets dominate sales. This setting best describes the National Football League (NFL), one of the four major North American leagues (NALs). Our chosen focus is on balance during the regular season and details of all our modeling choices are in the next section of the paper.

The quotes at the top of this paper help to illuminate the policy issue addressed by the theory. Is the NFL too balanced as Hall of Fame quarterback Troy Aikman’s quote at the top of this paper suggests? Or is the level of parity on the field somehow “optimal” as suggested in Commissioner Goodell’s quote? Congressional hearings (U.S. Senate, 2001) have even been convened on the subject in the remaining NALs.

To our reading of the policy-oriented literature, it is implicitly taken that more balance would be an improvement over the result generated by the leagues, themselves (comprehensive reviews are in Fort and Quirk, 1995; Szymanski, 2003; and Fort, 2006a). This debate—among fans, reporters, and economists—lacks anything remotely resembling any competitive balance target, e.g., an optimal level of competitive balance. From the perspective of optimality constructs, this intuitive belief that enhancing competitive balance would on net enhance general fan welfare may simply be Pareto non-comparable advocacy.

Rottenberg (1956) was the first to detail the problems associated with a lack of competitive balance. If outcomes on the field, court, or ice become too predictable, as when there are only a few very dominant teams, fans of perennially unsuccessful teams may stay away in droves and some teams in the league may actually go under. And even the teams that survive will have lower revenues if these disillusioned fans forsake the sport altogether. Thus, leagues have a vested interest in managing the level of competitive balance. The point of departure for this paper is how the league's profit-maximizing choice of competitive balance deviates from one specification of the social welfare-maximizing level of balance.

We address the question of optimal competitive balance comparing decentralized league outcomes to the level of competitive balance that maximizes the sum of consumers' and producers' surpluses. We realize that other possible Pareto optimal outcomes might be developed, but the surplus maximizing approach does have the virtue of utilizing theoretical tools that are conceptually amenable to relatively straightforward measurement and comparison in actual leagues. In particular, our main result is that whether the decentralized league result is too much or too little balance, relative to the surplus-maximizing level, is an empirical matter. If the marginal impacts of talent rearrangements are larger in smaller-revenue markets than in larger-revenue markets, then welfare is enhanced by rearranging talent to create more balance. But if the reverse is true, then less balance enhances social welfare. The elements required to actually assess this relationship are data and careful empirical analysis can settle the issue.

We also are not blind to the fact that some advocacy of particular mechanisms to enhance balance actually may be thinly veiled attempts to redistribute wealth from players to owners and among owners. But that is another virtue of our exercise. Future assessments of the distributional consequences of various approaches to competitive balance can now proceed with a firm grasp of the optimal target.

The paper proceeds as follows. In Section II, we compare the decentralized league model and planner's optimum for leagues that heavily utilize season ticket sales. Our analysis identifies the conditions that determine whether the planner would prefer more or less balance than this type of league will produce in its decentralized equilibrium. All of our findings suggest that whether more balance is preferred to less rests on empirical questions that have yet to be assessed and Section III lays out the policy implications. Conclusions and suggestions for future research round out the paper in Section V.

II. Optimal Competitive Balance in a "Season Ticket" League

In some leagues, season ticket sales dominate team revenue functions. For example, the NFL has only 8 home games and two preseason games to sell. Team Marketing Report (2008) tabulates average seat prices weighted by the proportions of different types of seats in stadiums for all NFL teams. The highest of these is about $118 per game suggesting a season ticket price in the neighborhood of $1,180. If the team performs below expectations, fans have only lost the value of the few games they then choose not to attend. So football owners are able to do what every owner would like to be able to do, namely, transfer the risk that the team performs below expectations to fans. Fans confronted primarily with season ticket options must make their estimate of the value of that purchase primarily on the quality of the home team. Further, in terms of post-season chances, every game in the NFL is more important to fans, even those against poorer opponents, dampening the importance of visiting team quality in the fan purchase decision. Our first modeling choice is to focus on a season ticket league and we will assume demand depends upon own ticket price and the team's own winning percent.

For our second modeling choice, we embed a season ticket league inside a broader "closed league," competitive talent equilibrium model (originally, El Hodiri and Quirk, 1971). Members of a closed league essentially face a completely inelastic supply of talent; "open league" members might increase their talent by importing it from some other league. The NFL, in particular, is distinguished on the closed league basis from other world leagues (e.g., world football). Recently, there has been some international talent migration in the other NALs.

The competitive talent market distinction is best portrayed by the classical Walrasian tatonnement referee. Using all information on the impacts of one team's talent choice on the other teams in the league, the referee's price comes to rest where no league member would change their talent choice. While we find this competitive process acceptable for our needs, especially for NALs, we note that the veracity of the competitive talent market choice is currently under contention (see Szymanski, 2004; Szymanski and Kesenne, 2004; Fort, 2006b; Fort and Quirk, 2007).

Our third modeling choice is to focus on regular season play. We further assume a league at a given absolute level of play, the major league level, and that all differences among teams at that level are relative differences (extensions are in Marburger, 1997; Rascher, 1997; and Kesenne, 2000). This builds Rottenberg's (1956) outcome uncertainty observation into the model since fans care about relative competition.

The remaining modeling choices are as follows. We restrict our attention to gate and attendance-related local revenue that can be portrayed as proportional to ticket price (Heilmann and Wendling, 1974). This abstracts from local TV revenue but, at least for the NFL, local TV is a relatively minor item in team revenues. We assume no team-specific contributions to the value of talent (Vrooman, 1996). Following the observations in Fort and Winfree (forthcoming), the marginal product of talent is assumed constant (constant returns to scale since talent is the long-run choice of team owners) so that characteristics of the underlying contest success function are essentially assumed away.

There is no need to delve into any mechanisms used to alter the league outcome, such as revenue sharing, in order to derive our comparisons between the league and the planner. The impacts of revenue sharing in a rational expectations equilibrium for our model, as well as for a league where single-game tickets dominate team revenues, are in Fort and Quirk (2007).

We adopt the following notation:

$I = \{1,...,n\}$ is an index of the set of teams in the league.

w_i = win percent of team i.

p = market price per unit of win percent.

$t_i(w_i)$ = ticket price at team i.

$D_i(t_i(w_i), w_i)$ = demand for tickets at team i.

MRP_i = marginal revenue product of a unit of win percent given a revenue maximizing choice of t_i.

And our assumptions are as follows:

Measurement Assumption: Talent used to produce win percent is measured so that adding one more unit of talent increases win percent by one unit. This assumption has two implications. First, the price of a unit of win percent, p, is also the price of a unit of talent. Second, the marginal revenue product of win percent, MRP_i, is also the marginal revenue product of talent, that is, the demand for talent.

Ticket Price Assumptions: $\frac{d t_i}{d w_i} > 0$; fans are willing to pay more for higher quality measured by win percent. Further, for any choice of w_i, $t_i(w_i)$ is chosen to maximize revenue for that w_i, implying $\frac{\partial D_i}{\partial t_i}\frac{t_i}{D_i} = -1$. We use $t_i(w_i)$ to denote the revenue maximizing value of t_i, given w_i. Note that this is *not* the same thing as maximizing revenue with respect to w_i.

Attendance Demand Assumptions: For any given w_i, $\frac{\partial D_i}{\partial t_i} < 0$ so that ticket demand slopes downward for any w_i; for any t_i, $\frac{\partial D_i}{\partial w_i} > 0$ with $\frac{\partial^2 D_i}{\partial w_i^2} < 0$ so that increased quality shifts demand to the right, but at a decreasing rate.

Talent Demand Assumption: For any t_i, $\frac{\partial MRP_i}{\partial w_i} < 0$; the demand for talent slopes downward to the right.

Given all of the groundwork, above, the profit function for team i is:

(1) $\pi_i = t_i D_i\left(t_i(w_i), w_i\right) - p\, w_i$, i = 1, …, n.

Note that, consistent with a season ticket league, we assume ticket demand depends upon own ticket price and the team's own winning percent. At a maximum of profits, the first-order conditions are:

$$(2)\quad \frac{d\pi_i}{dw_i} = t_i(w_i)\frac{\partial D_i(t_i(w_i), w_i)}{\partial w_i} - p = MRP_i - p = 0\,,\ i = 1,\ \ldots,\ n,$$

$$\text{and } \sum_{i=1}^{n} w_i = \frac{n}{2}\,.$$

Expression (2) shows that all team owners set the marginal revenue product of talent equal to the marginal cost of talent. In expression (2), even though t is a function of w, our Ticket Price Assumption (t is chosen to maximize revenue for any value of w so that $\frac{\partial D_i}{\partial t_i}\frac{t_i}{D_i} = -1$) ends up cancelling out all terms involving $\frac{dt}{dw}$.

Let $t_i^* = t_i(w_i^*)$, $D_i^* = D_i(t_i(w_i^*), w_i^*)$ and $MRP_i^* = MRP_i(t_i(w_i^*), w_i^*)$, i = 1,…, n, be the profit-maximizing ticket price, and ticket demand and talent demand evaluated at that price, respectively. There is a league profit-maximizing equilibrium at (t^*, w^*, p) if:

$$(3)\quad MRP_i^* - p = 0\,,\ i = 1,\ldots,\ n, \text{ and } \sum_{i=1}^{n} w_i^* = \frac{n}{2}\,.$$

Expression (3) dictates that, at a league profit-maximizing equilibrium, marginal revenue product of talent is equalized across the league, that is:

$$(4)\quad MRP_i^* - MRP_j^* = 0\,,\ \ i, j = 1,\ldots,\ n.$$

For what follows, we observe two implications from expression (4). First, given our Ticket Price Assumption, this equilibrium also has total revenue maximized for the league as a whole. We make use of this observation in our specification of the planner's optimum, shortly. Second, in our model, it is possible for league revenues to be maximized, consistent with (4) and the Ticket Price Assumption, at a constant vector $w^{*} = 0.500$, that is, a perfectly balanced league. However, (2) makes it clear that this can only occur if $MRP_i = MRP_j$ for all i, j = 1, …, n and for all values of w. As long as there is variation in talent demand, itself, the league cannot be perfectly balanced. We make headway at this point by imposing well-ordered variation in MRP_i across the teams in the league:

<u>Globally Invariant Drawing Power (GIDP) Assumption</u>: Assume the set of teams $I = \{1,...,n\}$ is listed in order of drawing power such that:

i. $i > j$, $w_i > w_j \Rightarrow D_i(t, w_i) > D_j(t, w_j)$ for any common $t \geq 0$;

ii. Let q_D be the number of tickets demanded. Then $i > j$,

$$w_i > w_j \Rightarrow D_i^{-1}(q_D, w_i) > D_j^{-1}(q_D, w_j) \text{ for any common } q_D.$$

Generally speaking, this is the well-known larger- and smaller-revenue market distinction common in the analysis of sports leagues. Under the Globally Invariant Drawing Power Assumption, team 1 occupies the largest-revenue market; team 2 is in the next largest-revenue market and so on down to team n. This seems reasonable especially over any relevant team or league planning horizon since the location of teams helps determine their drawing power and team location is completely in the hands of the league itself.

We first look at competitive balance in the decentralized profit-maximizing equilibrium of the season ticket league:

> Proposition 1: In an n-team season ticket league, with a competitive talent equilibrium, with globally invariant drawing power among teams, league revenues are maximized at (t^*, w^*) satisfying (4) if and only if $t_i^* > t_j^*$ and $w_i^* > w_j^*$, for all $i > j$.

Proposition 1 shows that the decentralized league equilibrium exhibits competitive imbalance with larger-revenue market teams winning more than smaller-revenue market teams and charging higher ticket prices. As discussed previously, the only time this will not be true is if (removing the GIDP Assumption) there are no larger- and smaller-revenue markets to begin with, that is, talent demand is identical in all markets.

We next consider the planner's optimum. For simplicity, we have the planner take the monopoly pricing power of each team as given. Let $C_i = \int_{t_i(w_i)}^{\infty} D_i(t_i(w_i), w_i)\,d\,t_i$ and $R_i = t_i(w_i) D_i(t_i(w_i), w_i)$ be fans' surpluses and owner's surplus in market i, respectively. We use revenues for owner surpluses since decentralized profit maximization by owners in a season ticket league leads to maximization of the league's total revenue anyway. The planner chooses the vector w (the distribution of talent and, hence, competitive balance) to maximize the sum of surpluses, accounting for the adding-up constraint.

The Lagrangean for this problem is $L = \sum_{i=1}^{n}(C_i + R_i) + \lambda\left(\sum_{i=1}^{n} w_i - \frac{n}{2}\right)$ and the first-order conditions are $\frac{\partial L}{\partial w_i} = \frac{\partial C_i}{\partial w_i} + \frac{\partial R_i}{\partial w_i} + \lambda$, i = 1,…, n, and $\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} w_i - \frac{n}{2} = 0$. Let $t'_i = t_i(w'_i)$, $D'_i = D_i(t_i(w'_i), w'_i)$ and $MRP'_i = MRP_i(t_i(w'_i), w'_i)$, i = 1,…, n, be the welfare-maximizing ticket price, and ticket demand and talent demand evaluated at that price, respectively. The two derivatives in the first-order condition with respect to w_i are:

$$(5)\quad \frac{\partial C_i}{\partial w_i} = \int_{t'_i}^{\infty} \frac{\partial D_i}{\partial w_i} dt_i + \left[\frac{\partial t_i}{\partial w_i}\Big|_{t_i=\infty} \right]\left[D_i\big|_{t_i=\infty} \right] - \frac{\partial t_i}{\partial w_i} D'_i$$

$$= \int_{t'_i}^{\infty} \frac{\partial D_i}{\partial w_i} dt_i - \frac{\partial t_i}{\partial w_i} D'_i .$$

$$(6)\ \frac{\partial R_i}{\partial w_i} = \frac{\partial t_i}{\partial w_i} D'_i + t'_i \frac{\partial D_i}{\partial w_i} = \frac{\partial t_i}{\partial w_i} D'_i + MRP'_i .$$

In (5), for the demand function determined by w'_i, the *quantity demanded* at $t_i = \infty$ must be zero, that is, $D_i|_{t_i=\infty} = 0$; charge an infinite price and in the limit sell zero tickets.

The first-order conditions then become:

$$(7)\quad \frac{\partial L_i}{\partial w_i} = \int_{t'_i}^{\infty} \frac{\partial D_i}{\partial w_i} dt_i - \frac{\partial t_i}{\partial w_i} D'_i + \frac{\partial t_i}{\partial w_i} D'_i + MRP'_i + \lambda$$

$$= \int_{t'_i}^{\infty} \frac{\partial D_i}{\partial w_i} dt_i + MRP'_i + \lambda,\ i = 1,\ldots, n, \text{ and } \frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} w_i - \frac{n}{2} = 0 .$$

Given this, (7) implies the following in the planner's equilibrium:

$$(8)\quad MRP'_i - MRP'_j = \int_{t'_j}^{\infty} \frac{\partial D'_j}{\partial w_j} dt_j - \int_{t'_i}^{\infty} \frac{\partial D'_i}{\partial w_i} dt_i ,\ i, j = 1,\ldots, n.$$

The first thing we observe is, as with our earlier observation for the league profit-maximizing outcome, it is possible for *welfare* to be maximized, consistent with (7) and the Ticket Price Assumption, at a constant vector $w^* = 0.500$, that is, a perfectly balanced league. Once again, identical demands allow this to happen since both the right-hand side of (8) vanishes and $MRP'_i = MRP'_j$ for all i, j = 1, …, n and for all values of w. However, as long as there is variation in talent demand, itself, the league cannot be perfectly balanced since the right-hand side of (8) cannot be zero.

However, unlike the league profit-maximizing outcome, adding the GIDP Assumption to the Talent Demand Assumption does not settle anything. Under those assumptions (the latter has declining MRP_i with respect to w_i), we can only find that $\frac{\partial D_i}{\partial w_i} < \frac{\partial D_j}{\partial w_j}$ in equilibrium; the marginal effect of an increase in talent *on ticket demand* is larger in the smaller-revenue market j. But this is not enough to settle the issue since the sign of the right-hand side of (8) also rests on *the range of integration of these demand effects*, that is, $t_j > t'_j$ and $t_i > t'_i$. Reasonably, while $t'_i > t'_j$, the sign of (8) rests on the magnitude of this difference.

So, the difference between the league revenue-maximizing equilibrium result in (4) and the planner's optimum rests on the right-hand side of (8). On the right-hand of (8), attendance for each team changes directly with a change in that team's own talent level and surpluses follow suit. But it is the size of the marginal fan surpluses with respect to talent, in the smaller-revenue market compared to the larger-revenue market, which determines whether an increase in balance is welfare enhancing. Without assuming the problem away, but with the virtue of highlighting what ultimately will be an empirical issue, all we can show is:

> Proposition 2: In an n-team season ticket league, with a competitive talent equilibrium, with globally invariant drawing power among teams, with a planner that maximizes the sum of consumers' and producers' surpluses, if $\int_{t'_j}^{\infty} \frac{\partial D'_j}{\partial w_j} dt_j - \int_{t'_i}^{\infty} \frac{\partial D'_i}{\partial w_i} dt_i > 0$ then welfare is maximized at $(t(w'), w')$ where 1) $t'_i > t'_j$ and $w'_i > w'_j$, for all $j \neq i$ and 2) $w'_i < w_i^*$ and $w'_j > w_j^*$, for all $j \neq i$.

Proposition 2 shows that the planner's equilibrium is also characterized by imbalance, but less so than the decentralized league equilibrium, *as long as the marginal fan surpluses with respect to talent are larger in the smaller-revenue market than the larger-revenue market* (that is, the right-hand side of (8) is positive). Otherwise, it could be the case that the league has chosen the welfare maximizing level of balance (the right-hand side of (8) is zero), or that a decrease in balance could be welfare enhancing (the right-hand side of (8) is negative).

Of course, the ultimate value of Proposition 2 is that it states the conditions under which an increase in balance will be welfare enhancing in the form of a testable hypothesis. A suitable statistical test is simply a test of whether the right-hand side of (8) is positive. Marginal impacts of winning percent on attendance can be obtained from estimating attendance demand. Since ticket prices are also available, the data required to do the test are observable.

III. Policy Implications

Starting from this basic welfare theory foundation, what can be said for policy? For the season ticket league, whether or not improved balance also improves welfare (defined as the sum of fans' and owners' surpluses) depends on the size of marginal consumers' surpluses in larger-revenue and smaller-revenue markets. The verdict of empirical work on this ambiguity will determine whether welfare will be improved by enhancing competitive balance, reducing it, or simply leaving it alone. This finding suggests that settling the debate over competitive balance requires knowledge of the relative sizes of changes in fan surpluses and ticket prices, between larger- and smaller-revenue markets, that accompany alterations in team qualities.

In essence, estimates of the impact of changes in quality on attendance actually measure the empirical importance of Rottenberg's (1956) uncertainty of outcome hypothesis; how does attendance respond to increases in a team's own quality? Estimates of those impacts to date have been somewhat clumsy and not clearly directed to the issues raised in Proposition 2, but they are evolving (the review in Szymanski, 2003, covers their findings while the review in Fort, 2006a, covers the shortcomings of the approaches). The analysis here dictates that these estimates are required *before* making any attempt to alter competitive balance.

To make these observations a bit more concrete, as mentioned earlier in the paper, some worry that the NFL may be too balanced. This view rejects Proposition 2 so that less balance would enhance fans' welfare. But this type of rejection is intuitive; only a careful econometric analysis of the direct effects stated in Proposition 2 would actually decide the issue.

While identification of mechanisms to either enhance or reduce competitive balance is out of place given our findings, our examination of optimal competitive balance does offer the following observation. In the event that league choices are judged sub-optimal, owners in NALs cannot be expected to violate their profit maximizing choices of their own volition. And there is no external regulatory agency to enforce the talent redistribution in a NAL that would have to occur. To date, there has only been Congressional brow-beating of the variety cited earlier in the paper. Thus, there is a regulatory structure hurdle to leap in order to move decentralized league decision making toward the welfare improvements under the planner's optimum.

From this perspective, one policy prescription—breaking up NALs into competitive separate leagues—has two attractive features to recommend it (see Horowitz, 1976; Noll, 1976; Ross, 1989, 1991; and Quirk and Fort, 1999). First, the structure for this type of intervention already exists under the antitrust laws. Fort (2007) lists the references in the argument over practical, case-by-case antitrust intrusions into sports, but the first important observation is that the structure for antitrust intervention is in place and breaking up production units has been accomplished in the past (for example, the AT&T break up into seven "baby Bells," finalized in 1982).

The second attractive feature of an antitrust move to break up sports leagues involves the chances for a Pareto improvement in fans' welfare. Proponents cited above have developed the argument that, if two competing leagues were created from an existing league, the result would unleash competitive forces so that a team would exist in every economically viable location without sacrificing the goodwill investment owners have already made in existing teams. If the result of such a break up approaches the competitive distribution of teams, Pareto optimality would reign over the distribution of talent among these teams with the sum of producers' and consumers' surpluses maximized. This would be the planner's outcome with optimal competitive balance detailed in this paper. Competition would distribute teams so that any remaining smaller- and larger-revenue potential among the franchises would approximate the optimal distribution of talent in the planner's outcome in (8) for the season ticket league.

To conclude our policy discussion, we are not so naïve as to believe that the sum of fan and owner welfare matters in the policy process. Indeed, public choice analysis often reveals that that social welfare may matter little in the policy process. Every move away from any league's profit maximizing choice will have distributional consequences on players, owners, and fans. Indeed, leagues have a variety of mechanisms for attaining their own ends and the ones chosen and favored by leagues must have favorable distributional consequences for them. But identifying the optimal level of competitive balance is important because doing so sets the stage for the analysis of the distributional consequences relative to welfare maximization; it is possible to know the cost in terms of fan welfare of violating the planner's optimum.

V. Conclusions and Suggestions for Further Research

We seek to remedy the absence of considerations of the optimal level of competitive balance in North American sports leagues. We devise a planner's optimal talent distribution for regular season play that maximizes the sum of fans' and owners' surpluses. That outcome is compared to decentralized, profit-maximizing outcome for a league where season ticket sales dominate team revenues (like in the NFL).

Except where all owners face identical demand functions, the profit-maximizing outcome yields competitive imbalance. Whether such a season ticket league has too little or too much balance depends on the marginal surpluses created in larger-revenue markets relative to smaller revenue markets following a planner's alteration in the distribution of talent. Theoretically, the possibility remains that the league's profit-maximizing distribution of talent also maximizes fan welfare.

For policy implications, first, only careful empirical tests can determine whether enhancing or reducing balance is welfare improving for our season ticket league. Careful estimates of impacts of talent choice on attendance demand for all teams are required in order to choose intervention mechanisms that effectively hit the optimal level of competitive imbalance. To date, this type of careful assessment is missing in the debate over competitive balance. Second, unless it also maximizes profits, we do not expect that owners will choose the optimal level themselves. Currently, there is no external regulatory structure governing NALs that could impose the planner's optimum using the variety of mechanisms that are capable of changing competitive balance. Third, the antitrust remedy of breaking up the leagues does already have the requisite legal structure and precedence. And if the forces of competition can drive a Pareto result, then the optimal level of balance that maximizes the sum of fans' and owners' surpluses, detailed in this paper, would be achieved.

There are many avenues for future work suggested by this analysis. We utilize the competitive talent market model most applicable to NALs. But initial investigations by Szymanski (2004), Szymanski and Kesenne (2004), and Easton and Rockerbie (2005) suggest that other leagues around the world may be better treated with non-cooperative models. And extensions beyond gate demand to include local TV will no doubt prove insightful for examples beyond the NFL. For that matter, different owner objectives would produce different decentralized league outcomes for comparison to the planner's optimum (most recently, Fort and Quirk, 2004; Kesenne, 2005). Further, we do not address the case where a league might be dominated by single-game ticket sales (more likely for, say, Major League Baseball). Finally, ours is an assessment of optimal balance during the regular season. While regular season balance bears directly on playoff accessibility, models of optimal playoff balance remain for future work.

Appendix: Proofs of Propositions

Proposition 1: We seek to show that, in equilibrium, $t_i^* > t_j^*$ and $w_i^* > w_j^*$.

For $w_i^* > w_j^*$, the GIDP Assumption has $MRP_i > MRP_j$ for any $0 \le w \le 1$ and for all $j \ne i$. Since our Talent Demand Assumption has $\frac{\partial MRP_i}{\partial w_i} < 0$, then $MRP_i > MRP_j$ for any $0 \le w \le 1$ and for all $j \ne i$ implies $w_i^* > w_j^*$.

Turning to $t_i^* > t_j^*$, expression (4), implies that $t_i^* \frac{\partial D_i^*}{\partial w_i} = t_j^* \frac{\partial D_j^*}{\partial w_j}$. Since $w_i^* > w_j^*$ implies $\frac{\partial D_i^*}{\partial w_i} < \frac{\partial D_j^*}{\partial w_j}$ then $t_i^* > t_j^*$ in order to satisfy (4).

Proposition 2: We seek to show that if $\int_{t'_j}^{\infty} \frac{\partial D'_j}{\partial w_j} d t_j - \int_{t'_i}^{\infty} \frac{\partial D'_i}{\partial w_i} d t_i > 0$ (the right-hand side of (8) is positive) then 1) $t'_i > t'_j$ and $w'_i > w'_j$, for all $j \ne i$ and 2) $w'_i < w_i^*$ and $w'_j > w_j^*$, for all $j \ne i$.

That $t'_i > t'_j$ and $w'_i > w'_j$, for all $j \ne i$ follows lines similar to the proof of Proposition 1, using the additional assumption that $\int_{t'_j}^{\infty} \frac{\partial D'_j}{\partial w_j} d t_j - \int_{t'_i}^{\infty} \frac{\partial D'_i}{\partial w_i} d t_i > 0$. For $w'_i > w'_j$, the GIDP Assumption has $MRP_i > MRP_j$ for any $0 \le w \le 1$ and for all $j \ne i$. Since our Talent Demand Assumption has $\frac{\partial MRP_i}{\partial w_i} < 0$, then $MRP_i > MRP_j$ for any $0 \le w \le 1$ and for all $j \ne i$ implies $w'_i > w'_j$. Turning to $t'_i > t'_j$, if

$\int_{t'_j}^{\infty} \frac{\partial D'_j}{\partial w_j} dt_j - \int_{t'_i}^{\infty} \frac{\partial D'_i}{\partial w_i} dt_i > 0$, then (8) implies that $t'_i \frac{\partial D'_i}{\partial w_i} > t'_j \frac{\partial D'_j}{\partial w_j}$. Since

$w'_i > w'_j$ implies $\frac{\partial D'_i}{\partial w_i} < \frac{\partial D'_j}{\partial w_j}$, then $t'_i > t'_j$ in order to satisfy (8) as long as the right-hand side of (8) is positive.

To see $w'_i < w_i^*$, if we evaluate the planner's optimum depicted in (8) at w*, then the planner's optimum is the same as the league profit-maximizing equilibrium if and only if $\int_{t_j^*}^{\infty} \frac{\partial D_j}{\partial w_j} dt_j - \int_{t_i^*}^{\infty} \frac{\partial D_i}{\partial w_i} dt_i = 0$. As noted in the proof of Proposition 1,

$\frac{\partial D_i^*}{\partial w_i} < \frac{\partial D_j^*}{\partial w_j}$. If this inequality held for $t_j > t_j^*$, $t_i > t_i^*$, then

$\int_{t'_j}^{\infty} \frac{\partial D_j}{\partial w_j} dt_j - \int_{t'_i}^{\infty} \frac{\partial D_i}{\partial w_i} dt_i > 0$, implying that (locally, at least) $w'_i < w_i^*$. Then

$\sum_{i=1}^{n} w_i = \frac{n}{2}$ implies $w'_j > w_j^*$, for all $j \neq i$. Moving from the league profit-maximizing equilibrium to a planner's optimum would be in the direction of more competitive balance.

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